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CBSE

CLASS-11th

THE CENTRAL BOARD OF SECONDARY EDUCATION

MATHS-II

SIERRA INNOVATIONS

CONTENT

PART - B

10. Straight line

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Straight line:

Definition:

Let us start our discussion with the very beginning by defining line.

A line is something which has no end points. It can be extended both the sides

I

If a straight line in the coordinate plane makes an angle with OX, then m=tan is called the slope or gradient of the line which can be zero, positive or negative. The slope of the X-axis and of the straight lines parallel to the X-axis will be zero. The slope of the Y-axis or of a line parallel to the Y- axis is undefined.

The slope of a straight line passing through two points (x_1,y_1) and (x_2.y_2) is y2-y1/x2-x1 and in particular slope of a line passing through the origin and the point (x,y) is y/x.

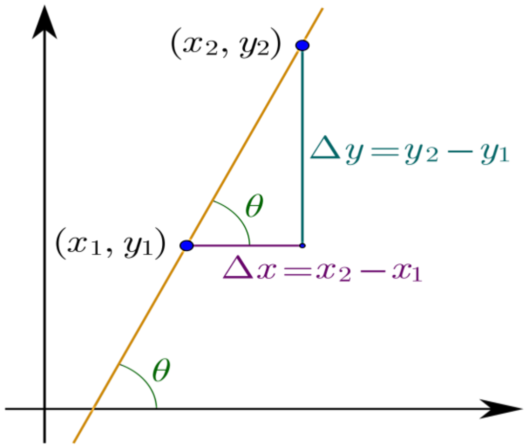


Fig 1.0 – Diagram of a staright line

STANDARD EQUATIONS:

- ✓ Equation of the X-axis is y=0.
- ✓ Equation of the Y-axis is x=0.
- ✓ Equation of a straight line parallel to the X-axis is y=c which lies above or below the X-axis according as c > 0 or c < 0 respectively.

- ✓ Equation of a straight line parallel to the Y-axis is x=k which lies to the right or to the left of the Y- axis according as k > 0 or k < 0 respectively.
- ✓ Equation of a straight line in the slope is y= mx+c , where m is the slope and c is the Y-intercept of the line. This line cuts the positive or the negative Y-axis at the point (0, c) according as the Y-intercept c is > 0 or < 0 respectively.
- ✓ Equation of a straight line in the intercept form is x/a+y/b=1, where a and b are called the X-intercept and the Y-intercept respectively. If a > 0, the line cuts the positive X-axis and if a < 0the line cuts the negative X-axis. Similarly the line cuts the positive or the negative Y-axis according as b > 0 or b < 0 respectively.
- ✓ Equation of a straight line in the normal form is xcos+ysin=p, p > 0;ON=p is the length of the normal (perpendicular from the origin on the line) which makes an angle with OX.
- ✓ Equation of a straight line in the two point form passing through the two given points (x1,y1) and (x_2,y_2) is $y-y_1/x-x_1=y_2-y_1/x_2-x_1$
- ✓ Equation of a straight line in the point -slope form is $y-y_1=m(x-x_1)$, where m is the slope of the line passing through the point (x_1,y_1).
- ✓ Equation of a straight line passing through the point of intersection of the straight lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ is $(a_1x+b_1y+c_1)+k(a_2x+b_2y+c_2) = 0$

TIP NOTE:

- ✓ Area of the triangle whose vertices are (x_1,y_1), (x_2,y_2),and (x_3,y_3) is -

A = ½ $\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

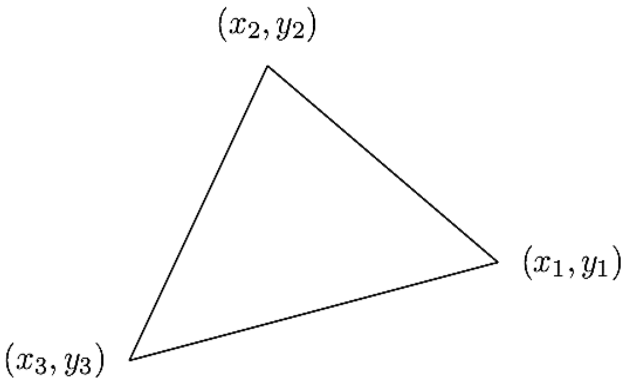


Fig 2.0 Area of triangle

- ✓ If the area of the triangle ABC is zero, then three points A,B and C lie on a line, i.e., they are collinear.

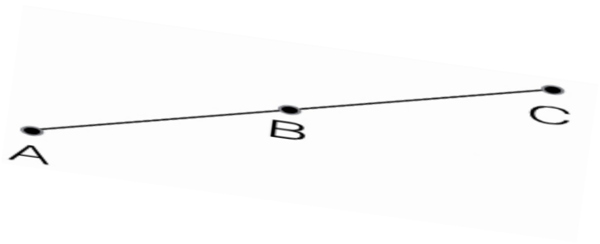


Fig 3.0 Area of triangle is zero

QUIZ: (Before checking the solution, first try it by yourself)

1. Find the equation of the line, which makes intercepts -3 and 2 on the x and y- axes respectively.

2. Equation a line is 3x - 4y + 10 = 0. Find its
   (i) slope,
   (ii) X and y-intercepts.

3. Show that two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where b_1,b_2 is not equal to (i.e.!=) 0 are:
   (i) Parallel if $a_1/b_1=a_2/b_2$,
   (ii) Perpendicular if $a_1a_2+b_1b_2=0$

<u>**Solutions:**</u>

1. Here, a=-3 and b=2. By intercept form, equation of the line is

X/ (-3) +y/ 2 = 1 or 2x -3y + 6 = 0

2. (i) Given, equation 3x-4y+10=0 can be written as-

y= (3/4) x + (5/2)

If we compare this with y=mx+c, we have slope of the given line as m=3/4

(ii) Equation 3x - 4y + 10 = 0 can be written as-

3x - 4y = -10 or x/ (-10/3) +y/ (5/2) =1

Here, a= -10/3 and y-intercept as b=5/2, if you compare it with the intercept equation.

3. Given, lines can written as-

$y=-(a_1/b_1)\,x-(c_1/b_1)$ ... (1)

$y=-(a_2/b_2)\,x-(c_2/b_2)$ ... (2)

Slopes of the lines (1) and (2) are $m_1 = -\ a_1/b_1$ and $m_2 = -\ a_2/b_2$, respectively. Now

(i) Lines are parallel, if $m_1=m_2$, which gives-

$-(a_1/b_1) = -(a_2/b_2)$ or $(a_1/b_1) = (a_2/b_2)$

(ii) Lines are perpendicular, if $m_1.m_2=-1$, which gives-

$(a_1/\ b1) * (a_2/b_2) = -1$ or $a_1a_2 + b_1b_2 = 0$

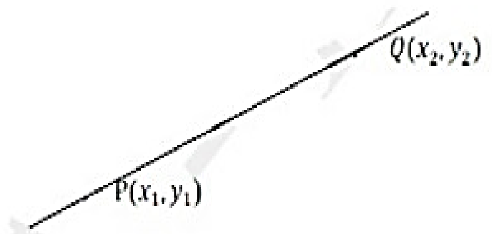
Slope of a line:

If θ is the inclination of a line l, then tan θ is called the slope or gradient of the

2. A line parallel to y – axis makes an angle of 90° with x – axis, so its slope is $\tan\frac{\pi}{2} = \infty$.

Slope of Line when Passing from two given points:

If $P(x_1, y_1)$ & (x_2, y_2) So, $m = \frac{y_2-y_1}{x_2-x_1}$



line l.

- The slope of a line whose inclination is 90° is not defined.
- The slope of a line is denoted by m.
- Thus, m = tan θ, θ ≠ 90°
- It may be observed that the slope of x-axis is zero and slope of y-axis is not defined.

Slope of a line when coordinates of any two points on the line are given:

slope of line l = m = tan θ. =(y2-y1)/(x2-x1)

Conditions for parallelism of lines in terms of their slopes:

Two non vertical lines l1 and l2 are parallel if and only if their slopes are equal.

m1 = m2.

tan α = tan β.

Conditions for perpendicularity of lines in terms of their slopes:

Two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other

m1 m2 = – 1.

Angle between two lines

<u>ANGLE BETWEEN TWO STRAIGHT LINES:</u>

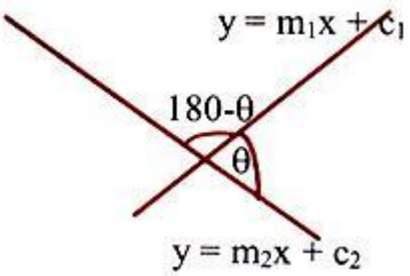


Fig 4.0 Angle between two straight lines

(1) The angle between the straight lines $y=m_1x+c_1$ and $y=m_2x+c_2$ is given by-
tan = $|m_2-m_1|/|1+m_2m_1|$ These lines are parallel if $m_1=m_2$ and perpendicular if $m_1m_2=-1$

(2) The angle between the straight lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ is given by

$tan=|a_1b_2-a_2b_1|/|a_1a_2+b_1b_2|$

QUIZ: (Before checking the solution, first try to do it yourself)

1. Line through the points (-2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Find the value of x ?

Solution:

Slope of the line through the points (-2,6) and (4,8) is-

$m_1= ((8-6)/(4-(-2)))=2/6=1/3$

Slope of the line through the points (8, 12) and (x, 24) is-

$m_2= ((24-12)/(x-8)) = 12/(x-8)$

Since two lines are perpendicular,m1.m2=-1, which gives-

(1/3)*(12/x-8) or x=4

Distance of a point from a straight line:

The distance of the point (x1,y1) from the line Ax+By+C=0, is

$|Ax_1+By_1+C|/|sqrt(A^2+B^2)|$

TIP NOTE:

When the point (x_1,y_1) and the origin lie on the opposite sides of the straight line Ax+By+C=0, then $d=(Ax_1+By_1+C)/sqrt(A^2+B^2)$ gives a positive value and when the point and origin lie on the same side of the line then d gives a negative value and that is why modulus sign is used for d.

The obtuse angle (say ϕ) can be found by using **ϕ =1800 – θ.**

Collinearity of three points:

three points are collinear if and only if slope of

AB = slope of BC.

ANGLE BISECTORS:

If straight lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ intersect, then the equations of the bisectors of the angles between the lines are-

$(a_1x + b_1y + c_1/ \text{sqrt} (a_1^2 + b_1^2)) = \pm (a_2x + b_2y + c_2/ \text{sqrt} (a_2^2 + b_2^2))$

CONCURRENCE OF THREE STRAIGHT LINES:

If the straight lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are concurrent; then $a_1 (b_2c_3 - b_3c_2) + b_1 (c_2a_3 - c_3a_2) + c_1 (a_2b_3 - a_3b_2) = 0$

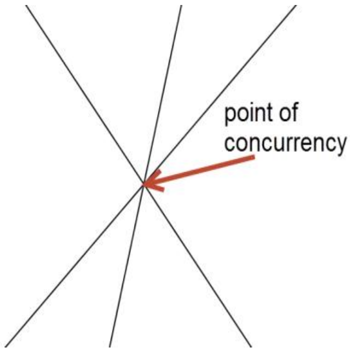


Fig 5.0 Point of concurrency

The condition can be expressed in terms of determinant as-

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

POSITION OF TWO POINTS WITH RESPECT TO A STRAIGHT LINE:

Let (x_1, y_1) and (x_2, y_2) be two given points and $ax+by+c=0$ be the equation of a given straight line.

- ✓ If ax_1+by_1+c and ax_2+by_2+c have the same sign, then the given points lie on the same side of the given line.
- ✓ If ax_1+by_1+c and ax_2+by_2+c have opposite sign, then the given points lie on the opposite sides of the given line.

Quiz: (Before checking the solution, try to solve it by yourself)

1. Find the equation of a line perpendicular to the line x-2y+3=0 and passing through the point (1,-2).
2. Find the distance between the parallel line 3x-4y+7=0 and 3x-4y+5=0

Solution:

1. Given, line x-2y+3 = 0 can be written as -

y= (1/2) x + (3/2)

Slope of the line (1) is m1=1/2. Therefore, slope of the line perpendicular to line (1) is -

m2= - (1/m1) = - 2

Equation of the line with slope -2 and passing through the point (1,-2) is-

Y - (-2) = -2 (x-1) or y = - 2x, which is the required equation.

2. Here, A=3, B=-4, C1=7 and C2=5,

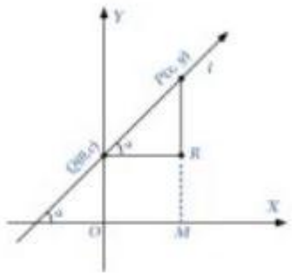
Therefore, the required distance is -

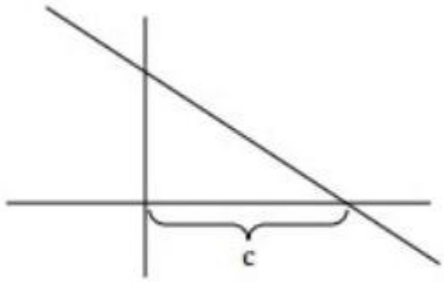
d=|7-5|/ sqrt (3^2 + (- 4)^2)|= 2/5

Different forms of the equation of a straight line:

1. **Slope intercept form of a line:**

   The equation of a line with slope m and making an intercept c on y – axis is y = mx + c





   The equation of a line with slope m and making an intercept c on x – axis is y = m(x - c)

2. **Point - slope form of a line:**

   The equation of a line which passes through the point (given) $P(x_1, y_1)$ and has the slope 'm' is

   $y - y_1 = m(x - x_1)$.

3. **Two point form of a line:**

   The equation of a line passing through two points $P(x_1, y_2)$ and $Q(x_2, y_2)$ is

   $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

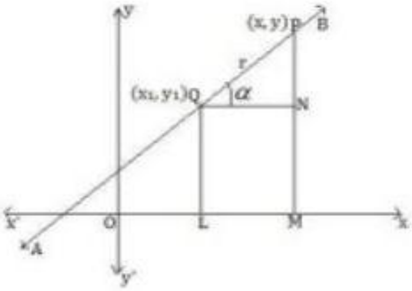
4. **Intercept form of a line:**

   The equation of a line which cuts off intercepts 'a' and 'b' respectively from the x – axis and y – axis is $\frac{x}{a} + \frac{y}{b} = 1$.

5. **Normal form or Perpendicular form of a line:**

   The equation of the straight line upon which the length of the perpendicular from the origin is p and this Perpendicular makes an angle α with x – axis is

6. **Distance form of a line:**
The equation of the straight line passing through (x_1, y_1) and making an angle θ with the +ve direction of x – axis is $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$
Where r is the distance of the point (x, y) on the line from the point (x_1, y_1)



Transformation of general equation in different standard forms:

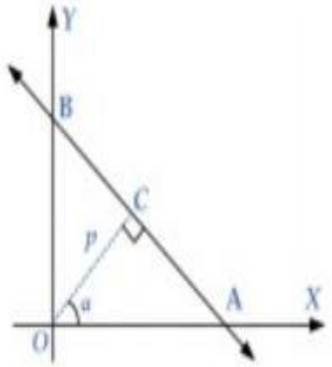
1. Transformation of Ax + By + C = 0 in the slope intercept form y = m x + c
$y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$
This is of the form y = m x +c, where
$m = -\frac{A}{B} = -\frac{cof\ ficient\ of\ x}{cof\ ficient\ of\ y}$, and intercept on y – axis $= -\frac{C}{B} = -\frac{constent}{cof\ ficient\ of\ y}$
2. Transformation of Ax + By + C = 0 in intercept form $\frac{x}{a} + \frac{y}{b} = 1$
$\frac{x}{\left(-\frac{C}{A}\right)} + \frac{y}{\left(-\frac{C}{B}\right)} = 1$
Intercept on x – axis $= -\frac{C}{A} = -\frac{cons\tan t\ term}{cof\ ficient\ of\ x}$, Intercept on y - axis $= -\frac{C}{B} = -\frac{cons\tan t\ term}{cof\ ficient\ of\ y}$
3. Transformation of Ax + By + C = 0 in intercept form $x\cos\alpha + y\sin\alpha = p$
$-\frac{A}{\sqrt{A^2+B^2}}x - \frac{B}{\sqrt{A^2+B^2}}y = \frac{C}{\sqrt{A^2+B^2}}$
Here $\cos\alpha = -\frac{A}{\sqrt{A^2+B^2}}$ and $\sin\alpha = -\frac{B}{\sqrt{A^2+B^2}}; p = \pm\frac{C}{\sqrt{A^2+B^2}}$

Distance between two parallel lines:

Step (i): Find the co-ordinates of any point on one of the given line, by putting x=0 and y=0

Step(ii): The perpendicular distance of this point from the other line is the required distance between the lines.

$x\cos\alpha + y\sin\alpha = p$



Concurrent Lines:

Three of more than three straight lines are said to be concurrent if they pass through a common point i.e., they meet at a point.

Condition of concurrency of three lines:

$a_1(b_2c_3 - b_3c_2) + b_1(c_2a_3 - c_3a_2) + c_1(a_2b_3 - a_3b_2) = 0$

EQUATIONS OF FAMILY OF LINES THROUGH THE INTERSECTION OF TWO LINES

$A_1x + B_1y + C_1 + k(A_2x + B_2y + C_2) = 0$

where k is a constant and also called parameter.

This equation is of first degree of x and y, therefore, it represents a family of lines.

DISTANCE BETWEEN TWO PARALLEL LINES

Working Rule to find the distance between two parallel lines:

(i) Find the co-ordinates of any point on one of ht egiven line, preferably by putting $x = 0$ and $y = 0$.

(ii) The perpendicular distance of this point from the other line is the required distance between the lines.

Questions and Answers

1 Mark Each:

1. Find the value of x for which the points (x, – 1), (2, 1) and (4, 5) are collinear.

Solution:

Here we have,

Points (x, – 1), (2, 1) and (4, 5) are collinear,

Slope of AB = Slope of BC

Then, (1+1)/(2-x) = (5-1)/(4-2)

2/(2-x) = 4/2

2/(2-x) = 2

2 = 2(2-x)

2 = 4 – 2x

2x = 4 – 2

2x = 2

x = 2/2

= 1

Thus, The required value of x is 1.

2. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points P (0, – 4) and B (8, 0).

Solution:

Here we have,

The co-ordinates of mid-point of the line segment joining the points P (0, – 4) and B (8, 0) are (0+8)/2, (-4+0)/2 = (4, -2)

The slope 'm' of the line non-vertical line passing through the point (x1, y1) and (x2, y2) is given by m = (y2 – y1)/(x2 – x1) where, x ≠ x1

The slope of the line passing through (0, 0) and (4, -2) is (-2-0)/(4-0) = -1/2

Thus,The required slope is -1/2.

3. Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right-angled triangle.

Solution:

Given,

The vertices of the given triangle are (4, 4), (3, 5) and (–1, –1).

The slope (m) of the line non-vertical line passing through the point (x_1, y_1) and (x_2, y_2) is given by $m = (y_2 – y_1)/(x_2 – x_1)$ where, $x \neq x_1$

So, the slope of the line AB (m_1) = (5-4)/(3-4) = 1/-1 = -1

the slope of the line BC (m_2) = (-1-5)/(-1-3) = -6/-4 = 3/2

the slope of the line CA (m_3) = (4+1)/(4+1) = 5/5 = 1

It is observed that, $m_1.m_3$ = -1.1 = -1

Hence, the lines AB and CA are perpendicular to each other

So,given triangle is right-angled at A (4, 4)

And the vertices of the right-angled Δ are (4, 4), (3, 5) and (-1, -1)

4. Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, -1).

Solution:

Here we have points A (5, 2), B (2, 3) and C (3, -1)

Firstly, we find the slope of the line joining the points (2, 3) and (3, -1)

Slope of the line joining two points $= \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{BC} = \frac{-1-3}{3-2} = -\frac{4}{1} = -4$

It is given that line passing through the point (5, 2) is perpendicular to BC

$\because m_1 m_2 = -1$

$\Rightarrow -4 \times m_2 = -1$

$\Rightarrow m_2 = ¼$

Therefore slope of the required line = ¼

Now, we have to find the equation of line passing through point (5, 2)

Equation of line: $y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \frac{1}{4}(x - 5)$

$\Rightarrow 4y - 8 = x - 5$

$\Rightarrow x - 5 - 4y + 8 = 0$

$\Rightarrow x - 4y + 3 = 0$

Hence, the equation of line passing through the point (5, 2) is $x - 4y + 3 = 0$

5. Find the Slope of a line which cuts off intercepts of equal lengths on the axes.

Solution:

We know that the equation of line in intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

Where a and b are the intercepts on the axis.

Given that a = b

$\Rightarrow \frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \frac{x+y}{a} = 1$

$\Rightarrow x + y = a$

$\Rightarrow y = -x + a$

$\Rightarrow y = (-1)x + a$

Since, the above equation is in y = mx + b form

So, the slope of the line is – 1.

6. Find the distance between P (x1, y1) and Q (x2, y2) when PQ is parallel to the y-axis,

Solution:

Here we have,