Toppersnotes

Unleash the topper in you

CBSE

CLASS-11th

THE CENTRAL BOARD OF SECONDARY EDUCATION

MATHS-I

SIERRA INNOVATIONS

CONTENT

PART – A

1. SET

CONTENT

Topics:

- **Set**
  - **Definition.**
  - **Symbol for special set.**
  - **Methods for Representation of set.**
- **Types of set.**
  - **The empty set**
  - **Finite set.**
  - **Infinite set**
  - **Equal set.**
  - **Non equal set.**
- **Subsets**
- **Power set**
- **Universal set**
- **Intervals as subsets of R**
- **Venn Diagram**
- **Union of sets**
- **Properties of operation of union**
- **Intersection of sets**
- **Properties of operation of intersection**
- **Difference of sets**
- **Complement of set**
- **Properties of complement o set**
- **Points to remember**
- **Question & answer**

SET

Definition:

A set is a well-defined collection of objects of a particular kind.

Ex.- Even natural numbers less than 10, i.e., 2, 4, 6, 8

Prime factors of 21, namely 3 and 7

The solution of the equation: $x^2 - 5x + 6 = 0$, viz, 2 and 3.

Symbol for special set:

N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

Z+ : the set of positive integers

Q+ : the set of positive rational numbers, and

R+ : the set of positive real numbers.

Notes:

(i) Objects, elements and members of a set are synonymous terms.

(ii) Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.

(iii) The elements of a set are represented by small letters a, b, c, x, y, z, etc.

Methods of Representation of set:

(i) Roster or tabular form: In this form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }.

Ex. The set of even natural numbers is represented by {2, 4, 6, . . .}. The dots indicates that the list of even numbers continue indefinitely.

Notes:

(i) The order of the listed elements does not make sense in Roster form. In case of above example we can also write {4, 2, 6, 8, ….)

(ii) Generally element is not repeated. For example, the set of letters forming the word 'COLLEGE' is { C, O, L, E, G} or {G, E, L, O, C}. Here, the order of listing elements has no relevance.

(ii) Set-builder form: In this form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

Ex. In the set {2, 4, 6, 8,}, all the elements possess a common property, namely, each of them is a even natural number.

V = {x : x is a even natural number.}

Types of set:

The Empty Set: The set having no elements in it, is called an empty set.

Symbol: ϕ or { }.

Ex. A = { x : x is a whole number greater than 1}

Finite set: A set which is empty or consists of a definite number of elements is called finite.

Ex. Let S be the set of solutions of the equation x^2 –25 = 0. Then S is finite.

Infinite set: The set other than finite set is called Infinite set.

Ex. Let P be the set of points on a line. Then P is infinite.

Note: All infinite sets cannot be described in the roster form

Equal Sets:

Let's consider two sets A and B, if every element of A is also an element of B and if every element of B is also an element of A, then the sets A and B are said to be equal. Clearly, the two sets have exactly the same elements.

Ex. Let A = {8, 2, 3, 4} and B = {3, 8, 4, 2}. Then A = B.

Non Equal Sets: The sets other than equal are known as Non Equal set.

Ex. Let A = {8, 5, 3, 4} and B = {3, 8, 4, 2}. Then A ≠ B.

Subsets:

Definition: A set X is said to be a subset of a set Y if every element of X is also an element of Y.

$$X \subset Y \text{ if } x \in X \Rightarrow x \in Y$$

Power Set: The collection of all subsets of a set X is called the power set of X. It is denoted by P(X). In P(X), every element is a set.

Ex. if X = { 3, 5 }, then

P(X) = { ϕ,{ 3 }, { 5 }, { 3,5 }}

Note: if X is a set with n(X) = m, then, n [P(X)] = 2^m.

Universal set:

The set which contains every single set or elements in it. It is denoted by U.

Ex. Let A={x:x is a student of 11a}

B={x:x is a student of 11b}

C={x:x is a student of 11c}

Then, U = {x:x is a student of 11}

Intervals as subsets of R: Lets consider a, b ∈ R and a < b.

Close interval: The interval which contains the end points also is called closed interval. It is denoted by [a, b].

[a, b] = {x : a ≤ x ≤ b}

Open interval: The interval which are closed at one end and open at the other, is called open interval.

Ex.

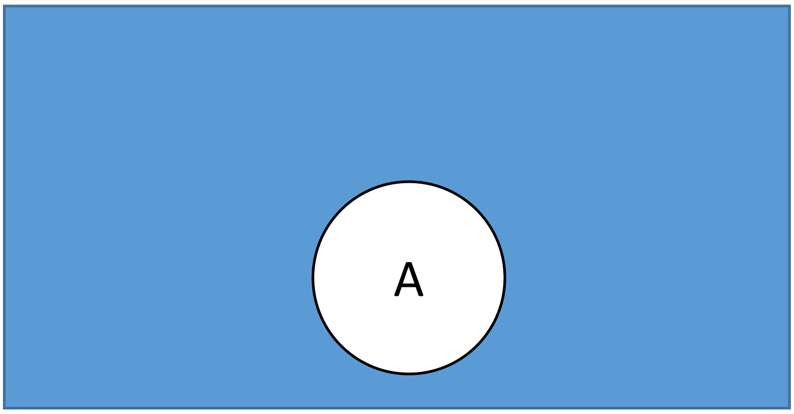
[a, b) = {x : a ≤ x < b} is an open interval from a to b, including a but excluding b.

(a, b] = { x : a < x ≤ b } is an open interval from a to b including b but excluding a.

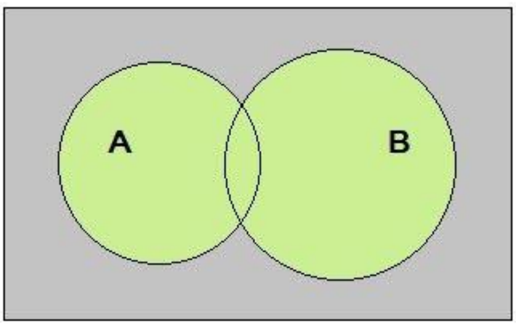
Venn Diagram:

The diagrams which consist of rectangles and closed curves
usually circles, are called Venn Diagram.
The universal set is represented usually by a rectangle and its subsets by Circles.
Here, we have U as universal set and set A is subset of it.

U



Union of sets: Lets consider A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once. The symbol 'U' is used to denote the union. Symbolically, we write A ∪ B and usually read as 'A union B'



Properties of the Operation of Union:

(i) A ∪ B = B ∪ A (Commutative law)

(ii) (A ∪ B) ∪ C = A ∪ (B ∪ C)

(Associative law)

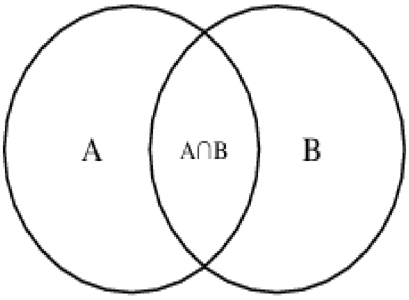
(iii) A ∪ ϕ = A (Law of identity element, ϕ is the identity of ∪)

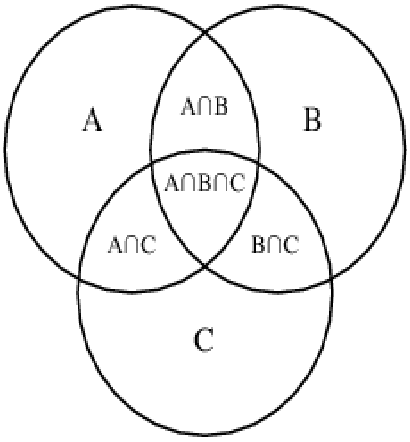
(iv) A ∪ A = A (Idempotent law)

(v) U ∪ A = U (Law of U)

<u>Intersection of set:</u> The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write

A ∩ B = {x : x ∈ A and x ∈ B}





Properties of Operation of Intersection:

(i) A ∩ B = B ∩ A

(Commutative law).

(ii) (A ∩ B) ∩ C = A ∩ (B ∩ C)

(Associative law).

(iii) ϕ ∩ A = ϕ, U ∩ A = A

(Law of ϕ and U).

(iv) A ∩ A = A

(Idempotent law)

(v) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) (Distributive law) i. e.,

∩ distributes over

Difference of sets: The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write A – B and read as " A minus B".

Complement of a set: Lets consider U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A′ to denote the complement of A with respect to U.

Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$. Obviously $A' = U - A$

Properties of Complement Sets:

1. Complement laws:

(i) $A \cup A' = U$

(ii) $A \cap A' = \phi$

2. De Morgan's law:

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

4. Laws of empty set and universal set $\phi' = U$ and $U' = \phi$

Points to Remember:

- A set is a well-defined collection of objects.
- A set which does not contain any element is called empty set.
- A set which consists of a definite number of elements is called finite set, otherwise, the set is called infinite set.
- Two sets A and B are said to be equal if they have exactly the same elements.
- A set A is said to be subset of a set B, if every element of A is also an element of B. Intervals are subsets of R.
- A power set of a set A is collection of all subsets of A. It is denoted by P(A).
- The union of two sets A and B is the set of all those elements which are either in A or in B.
- The intersection of two sets A and B is the set of all elements which are common. The difference of two sets A and B in this order is the set of elements which belong to A but not to B.
- The complement of a subset A of universal set U is the set of all elements of U which are not the elements of A.
- For any two sets A and B, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
- If A and B are finite sets such that $A \cap B = \phi$, then

$n(A \cup B) = n(A) + n(B)$.

If $A \cap B \neq \phi$, then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Questions & Answers:

1 Mark Each:

1. Find the solution set of the equation x ^2 + x – 2 = 0 in roster form.

Solution: The given equation can be written as

(x – 1) (x + 2) = 0, i. e., x = 1, – 2

Thus, the solution set of the given equation can be written in roster form as {1, – 2}.

2. Write the set {1/2,2/3,3/4,4/5,5/6,6/7} in the set-builder form.

Solution: Here, we have each member in the given set has the numerator one less than the denominator. Also, the numerator begin from 1 and do not exceed 6. Hence, in the set-builder form the given set is

{x : x = n/(n+1), where is a natural number and 1 ≤ n ≤ 6.}

3. Determine either the sets are finite or infinite.

A= {1, 2, 3 ...}

Solution: {1, 2, 3 …} is an infinite set because it has infinite number of natural numbers

3. Determine either the sets are Equal or not.

A = {x: x is a letter in the word FOLLOW}; B = {y: y is a letter in the word WOLF}

Solution: A = {x: x is a letter in the word FOLLOW} = {F, O, L, W}

B = {y: y is a letter in the word WOLF} = {W, O, L, F}

Order in which the elements of a set which are listed is not significant.

Therefore, A = B.

4. Examine whether the statement is true or false:

{a, e} ⊂ {x: x is a vowel in the English alphabet}

Solution:

True.

We know that a, e are two vowels of the English alphabet.

5. Write down all the subsets of the following sets:

{1, 2, 3}

Solution:

Subsets of {1, 2, 3} are

Φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, and {1, 2, 3}.

6. State if the given set is finite or infinite

Solution: A= { x : x ∈ **Z and** x x 8 **}** 2 − 9 + 1 = 0

Solution: Given condition, x x 8 2 − 9 + 1 = 0

Solve for x here,

x x x 8 2 − 3 − 6 + 1 = 0

(x − 3)(x − 6) = 0

x = 3, 6

7. State with reasons which of the following sets is an/ are empty set.

(i) Set of even prime numbers greater than 2

Solution: (i) Since 2 is the only even prime number, therefore it is an empty or null set.

8. State whether the given pairs of sets are equal or not.

(i) A={ -7, 5 } B={ x : x ∈ **Z and** x 2x 5 **}**

Solution : (i) Given A={-7,5}

For set B , we have solution of x 2x 5 as 7 and 5, 2 − 1 + 3 = 0

Therefore we have B={7,5} ⇒**A** =/ **B**

9. Write down all the subsets and power set of set A={1, 2, 3}

Solution: n(A) = 3 , total number of subsets = 23 = 8

Subsets of A={1},{ 2},{ 3}, {1, 2},{1,3},{2,3}, {1, 2, 3}, Φ Powerset of A, P(A)={ {1},{ 2},{ 3}, {1, 2},{1,3},{2,3}, {1, 2, 3}, Φ }

10. How many elements will a power set of A has, if A= Φ

Solution: We know that,

If n(A)=m then,

n{P(A)}= 2m

Since A is a null set, n(A)=0 ⇒ n{P(A)}= 20 = 1

Powerset of will have 1 element that is Φ .

2 Marks Each:

1. How many elements has P (A), if A = Φ?

Solution:

If A is a set with m elements

n (A) = m then n [P (A)] = 2m

If A = Φ we get n (A) = 0

n [P(A)] = 20 = 1

Therefore, P (A) has one element.

2. Write the following as intervals:

(i) {x: x ∈ R, 0 ≤ x < 7}

(ii) {x: x ∈ R, 3 ≤ x

Solution:

(i) {x: x ∈ R, 0 ≤ x < 7} = [0, 7)

(ii) {x: x ∈ R, 3 ≤ x ≤ 4} = [3, 4]

3. Write the following intervals in set-builder form:

(i) (6, 12]

(ii) [–23, 5)

Solution:

(i) (6, 12] ={x: x ∈ R, 6 < x ≤ 12}

(ii) [–23, 5) = {x: x ∈ R, –23 ≤ x

4. What universal set (s) would you propose for each of the following:

(i) The set of right triangles

(ii) The set of isosceles triangles

Solution:

(i) Among the set of right triangles, the universal set is the set of triangles or the set of polygons.

(ii) Among the set of isosceles triangles, the universal set is the set of triangles or the set of polygons or the set of
two-dimensional figures.

5. Let A = {a, b}, B = {a, b, c}. Is A ⊂ B? What is A ∪ B?

Solution:

It is given that

A = {a, b} and B = {a, b, c}

Yes, A ⊂ B

So the union of the pairs of set can be written as

A∪ B = {a, b, c} = B

3 Marks Each:

1. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}; find

(i) A ∪ B ∪ C

(ii) A ∪ B ∪ D

(iii) B ∪ C ∪ D

Solution:

(i) A ∪ B ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}

(ii) A ∪ B ∪ D = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(iii) B ∪ C ∪ D = {3, 4, 5, 6, 7, 8, 9, 10}

2. Which of the following pairs of sets are disjoint.

(i) {1, 2, 3, 4} and {x: x is a natural number and 4 ≤ x ≤ 6}

(ii) {a, e, i, o, u}and {c, d, e, f}

(iii) {x: x is an even integer} and {x: x is an odd integer}

Solution:

(i) {1, 2, 3, 4}

{x: x is a natural number and 4 ≤ x ≤ 6} = {4, 5, 6}

So we get

{1, 2, 3, 4} ∩ {4, 5, 6} = {4}

Hence, this pair of sets is not disjoint.

(ii) {a, e, i, o, u} ∩ (c, d, e, f} = {e}

Hence, {a, e, i, o, u} and (c, d, e, f} are not disjoint.

(iii) {x: x is an even integer} ∩ {x: x is an odd integer} = Φ

3. If X and Y are two sets such that X ∪Y has 18 elements, X has 8 elements and Y has 15 elements; how many elements does X ∩ Y have?

Solution:

Here we have,

n (X U Y) = 18

n (X) = 8

n (Y) = 15

We can write it as

n (X ∪ Y) = n (X) + n (Y) – n (X ∩ Y)

Substituting the values

18 = 8 + 15 - n (X ∩ Y)

By further calculation

n (X ∩ Y) = 23 – 18 = 5

So we get

n (X ∩ Y) = 5

4. Let A={ a, b, c, d} , B={ a, b,c } and C={b,d}. Find all sets D which satisfies the given conditions.

(i) D⊂ B and D⊄C

(ii) D ⊂ B, D = / **B and D⊄C**

(iii) D ⊂ A, D ⊂ B and D ⊂ C

Solution: (i) First we write all the subsets of B

P(B)={ {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}, Φ }

Similarly all the subsets of C,

P(C) ={ {b}, {d}, {b,d}, Φ }

Now P is a subset of B but not C

⇒ D ∈ P(B) and D ∈/P(C)

⇒D = {a},{c}, {a,c}, {a,b},{b,c}, {a,b,c}

(ii) D is a subset of B but notC and also it not equal to B

⇒ D ∈ P(B) , D =/ B and D ∈/P(C)⇒D = {a}, {c},{a,c}, {a,b},{b,c}

(iii) Here first write all the subset of A

P(A) = { {a}, {b}, {c}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,c}, {b,c,d}, {a,c,d}, {a,b,d},{a,b,c,d}, Φ } D is subset of A, B and C

⇒ D ∈ P(A) , D ∈ P(B) and D ∈ P(C)