Toppersnotes
Unleash the topper in you

CA FOUNDATION

The Institute of Chartered Accountants of India

BUSINESS MATHEMATICS & LOGICAL REASONING

SIERRA INNOVATIONS
Private Limited

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BUSINESS MATHEMATICS

LOGICAL REASONING

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RATIO & PROPORTION

A ratio is a Comparison of the size of two or more quantities of the Same kind by division.

If a and b are two quantities of the same kind (in Same unit), then the fraction a/b is Called the ratio of a to b.

★ It is written as a : b

★ Thus the ratio of a to b = a/b or a : b

★ The quantities a and b are Called the terms of the ratio.

★ a is Called the first term or antecedent and b is Called the Second term or Consequent.

Important notes:

(1) Both terms of a ratio Can be multiplied or divided by the Same (non-zero) number.

(2) The Order of the terms in ratio is important.

$$3 : 4 \neq 4 : 3$$

(3) Ratio exists Only b/w quantities of the Same kind.

(4) Quantities to be Compared must be in the Same kind.

(5) To Compare two ratio, Convert them into equivalent like fractions.

(6) If a quantity increases or decreases in the ratio a:b then new quantity = $\frac{b}{a}$ of the original quantity.

→ The fraction by which the original quantity is multiplied to get a new quantity is Called the factor multiplying ratio.

1.1.3 Inverse ratio:

One ratio is the inverse of another it their product is 1. Thus a:b is the inverse of b:a and Vice-versa.

1. A ratio a:b is Said to be of greter inequality if a>b and of less inequality if a>b.

2- The ratio Compounded of the two ratios a:b and c:d is ac:bd.
For example Compound ratio of 3:4 and 5:7 is 15:28.
Compound ratio of 2:3, 5:7 and 4:9 is 40:189.

3- A ratio Compounded of itself is Called its duplicate ratio. Thus $a^2 : b^2$ is the duplicate ratio of a:b Similarly, the triplicate ratio of a:b is $a^3 : b^3$.

For example, duplicate ratio of 2 : 3 is 4 : 9 Triplicate ratio of a : b is a 2 : 3 is 8 : 27.

4 ⇒ The Sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$ and the Sub triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$.
For example Sub-duplicate ratio of 4 : 9 is $\sqrt{4} : \sqrt{9} = 2 : 3$
And Sub-triplicate ratio of 8 : 27 is $\sqrt[3]{8} : \sqrt[3]{27} = 2 : 3$

5 ⇒ If the ratio of two Similar quantities Can be expressed as a ratio of two integers, the quantities are Said to be Commensurable, Otherwise, they are Said to be incommensurable. $\sqrt{3} : \sqrt{2}$ Cannot be expressed as the ratio of two integers and therefore, $\sqrt{3}$ and $\sqrt{2}$ are incommensurable quantities.

6 ⇒ Continued Ratio is the relation (or Comparison) between the magnitudes of three or more quantities of the Same kind, The Continued ratio of three quantities a, b, c is Written as a : b : c.

An equality of two ratios is Called a Proportion.

⇒ Four quantities a, b, c, d are Said to be in Proportion if $a:b = c:d$

⇒ $a:b :: c:d$

⇒ The quantities a, b, c, d are Called terms of the Proportion a, b, c and Called, it's first, second third and fourth terms respectively.

⇒ First and fourth terms are Called extremes.

⇒ Second and third terms are Called means.

⇒ If $a:b = c:d$ then d is Called fourth Proportional

⇒ If $a:b = c:d$ are in Proportion then $a/b = c/d$

⇒ $ad = bc$

⇒ Product of extremes = Product of means

(This is Called Cross product rule)

⇒ Three quantities a, b, c of the Same kind ar Said to be in Continous proportion.

$a:b = b:c \Rightarrow b^2 = ac$

⇒ If a, b, C are in Continous proportion, then the middle term b is Called the mean proportional b/w a & C a is the first proportional and C is the third Proportional

⇒ Thus if b is mean proportional b/w a & C, then $b^2 = ac$

⇒ Three or more numbers are related that the ratio of the first to the Second, the ratio of the third, third to the fourth, are all equal, the numbers are Said to be in Continued Proportion

$$\frac{x}{y} = \frac{y}{z} = \frac{z}{w} = \frac{w}{p} = \frac{p}{q} ------$$

When x, y, z, w, p, q are in Continued Proportion.

1.2.1 Properties of Proportion –

1 – if $a : b = c : d$, then $ad = bc$

Proof. $\frac{a}{b} = \frac{c}{d}$; $\therefore$ $ad = bc$ (By Cross-multiplication

2 – if $a : b = c : d$, then $b : a = d : c$ (Invertendo)

Proof. $\frac{a}{b} = \frac{c}{d}$ or $1 \Big/ \frac{a}{b} = 1 \Big/ \frac{c}{d}$, or, $\frac{b}{a} = \frac{d}{c}$

3 – if $a : b = c : d$, then $a : c = b : d$ (Alternendo)

Proof. $\frac{a}{b} = \frac{c}{d}$ or, $ad = bc$

Dividing both Sides by cd, we get.

$\frac{ad}{cd} = \frac{bc}{cd}$, or $\frac{a}{c} = \frac{b}{d}$, i.e $a : c = b : d$

4 – if $a : b = c : d$, then $a+b ; b = c+d : d$ (Componendo)

Proof. $\frac{a}{b} = \frac{c}{d}$, or, $\frac{a}{b} + 1 = \frac{c}{d} + 1$

or, $\frac{a+b}{b} = \frac{c+d}{d}$, i.e $a+b ; b = c+d : d$.

5 – if $a : b = c : d$, then $a-b.b = c-d : d$ (Dividendo)

Proof $\frac{a}{b} = \frac{c}{d}$, or $\frac{a}{b} + 1 = \frac{c}{d} + 1$, or

$\frac{a+b}{b} = \frac{c-b}{d}$, i.e. $a-b ; b = c-d ; d$.

6 ⇒ if $a:b = c:d$, then $a+b : a-b = c+d : c-d$ (Componendo and Dividendo)

Proof, $\frac{a}{b} = \frac{c}{d}$, or $\frac{a}{b} + 1 = \frac{c}{d} + 1$, or

$$\frac{a+b}{b} = \frac{c+d}{d} \quad \text{---------------} \quad 1$$

Again, $\frac{a}{b} - 1 = \frac{c}{d} - 1$, or $\frac{a-b}{b} = \frac{c-d}{d}$ ------------ 2

Dividing (1) by (2)

We get,

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ i.e $a+b : a-b = c+d : c-d$

(7) if $a:b = c:d = e:f = \text{----------}$ then

each of these rations is equal

$(a+c+e+\text{-------}) : (b+d+f+\text{--------})$

SURDS & INDICES

The index of a number Says how many times to use the number in a multiplication

⇒ It is Written as a Small number to the right and above the base number.

Ex:-

8^{2}

2 → exponent (or) Index (or) Power

Base → 8

means,
A factor which multiples is Called the "base" and the number of times it is multiplied is Called the "Power" or the "index".

Law I

$a^m \times a^n = a^{m+n}$

when m and n are Positive integers.

Ex:- $3^4 \times 3^5$
⇒ $3^{4+5} = 3^4$.

Law II $\frac{a^m}{a^n} = a^{m-n}$, when m and n are positive integers.

Ex:- $\frac{2^7}{2^4} = 2^{7-4} = 2^3 = 8$.

Law-3

$$(a^m)^n = a^{mn},$$

where m and n are positive integers.

Example:

$$(2^4)^3 = 2^{4\times 3} = 2^{12} = 4096.$$

Law-4

$$(ab)^n = a^n b^n$$

When n can take all of the values.

Example:

$$6^3 =$$
$$= (2\times 3)^3 = 2^3 \times 3^3 = 8\times 27 = 216.$$

Properties -

① $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$

② $a^x = a^y$ then $x = y$

③ $x^a = y^a$ then $x = y$

④ $\sqrt[m]{a} = a^{1/n}$

LOGARITHM

The logarithm is the increase function to exponentiation

⇒ That means the logarithm of a given number x is the exponent to which another fixed number the base b, must be raised, to produce that number x.

Ex:- how many 2s we multiply to get 8?

⇒ $2 \times 2 \times 2 = 8$, So we had to multiply is called the 'base'

⇒ " the logarithm of 8 with base 2 is 3.

⇒ or " log base 2 of 8 is 3"

or

" the base 2 log of 8 is 3.

Example:- $2^4 = 16$

$\log_2 16 = 4$

Facts :-

1 ⇒ The two equation $a^x = n$ and $x = \log_a n$ are Only transformation of each other and should be remembered to change One form of the relation into the other.

2 ⇒ The logarithm of 1 to any base is zero, This is because any number raised to the power zero is One.
Since $a^0 = 1$, $\log_a 1 = 0$

3 ⇒ The logarithm of any quantity of the Same base is unity. This is because any quantity
Since $a^1 = a$, $\log_a a = 1$

Examples :-

1. if $\log_a \sqrt{2} = \frac{1}{6}$, find the Value of a.

~~2.~~ We have $a^{1/6} = \sqrt{2} \Rightarrow a = (\sqrt{2})^6 = 2^3 = 8$

2- Find the logarithm of 5832 to the base $3\sqrt{2}$

Let us take $\log_{3\sqrt{2}} 5832 = x$

We may Write, $(3\sqrt{2})^x = 5832 = 8 \times 729 = 2^3 \times 3^6 = (\sqrt{2})^6 (3)^6 = (3\sqrt{2})^6$

Hence, $x = 6$

Logarithms of numbers to the base 10 are known as Common logarithm.

1.4.1 Fundamental Laws of Logarithm.

1⇒ Logarithm of the Product of two numbers is equal to the Sum of the Logarithms of the numbers to the Same base i.e.

$$\log_a mn = \log_a m + \log_a n$$

2⇒ The logarithm of the quotient of two numbers is equal to the difference of their logarithm to Same base.

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

3⇒ logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base.

$$\log_a m^n = n \log_a m$$

Change of base

If the logarithm of a number to any base is given then the logarithm of the same number to any other base can be determined from the following relation

$$\log_a m = \log_b m \times \log_a b$$

$$\Rightarrow \log_b m = \frac{\log_a m}{\log_a b}$$

Ex:- $\log_3 8 = \frac{\log_{10} 8}{\log_{10} 3} = \frac{\log_{10} 2^3}{\log_{10} 3} = \frac{3\log_{10} 2}{\log_{10} 3}$

Logarithm tables:-

The logarithm of a number Consists of two parts, the whole part or the integral part is Called the Chracteristic and the decimal part is Called the "mantissa".
Where the former Can be known by more insepection the letter has to be Obtained from the logarithm tables.

Characteristic:-

The Chracteristic of the logarithm of any number greater than, is positive and is One less than the Number of digits to the left of the decimal point in the given number.

Zero on positive Chracteristic When the number under Consideration is greater than unity.

Since - $10^0 = 1$, $\log 1_{10} = 0$

$10^1 = 10$, $\log 10 = 1$

$10^2 = 100$ $\log 100 = 2$

$10^3 = 1000$ $\log 1000 = 3$

negative Chracteristics -

$10^{-1} = \frac{1}{10} = 0 = 1 \rightarrow \log 0.1 = -1$

$10^{-2} = \frac{1}{100} = 0.01 \rightarrow \log 0.01 = -2$

Mantissa

The mantissa is the fractional part of the logarithm of a given number -

Number	Mantissa	logarithm
log 4594	= (-----6623)	= 3·6623
log 459·4	= (----6623)	= 2·6623
log 45·94	= (-------6623)	= 1·6623
log 4·594	= (---------6623)	= 0·6623
log ·4594	= (--------6623)	= $\bar{1}$·6623

Anti logarithm

If x is the logarithm of a given number n with a given base then n is called antilogarithm of x to that base.

Ex:- $\log_a n = x$ then $n = \text{antilog } x$

Number	Logarithm
206	2·3139
20·6	1·3139
2·06	0·3139
·206	−1·3139
·0206	−2·3139

Relation b/w indics and logarithm

① $\log_a^{m} + \log_a^{m} = \log_a mn$

② $\log_a m^n = n \log_a^{m}$

③ $\log_b^{a} \times \log_a^{b} = 1$

④ $\log_b c \times \log_c b = 1$

Important hints

- $\text{Log}, mn = \log_a m + \log_a n$

  Ex, $\log(2\times3) = \log^2 + \log 3$

- $\log_a(m/n) = \log_a m - \log_b n$

  Ex- $\log(3/2) = \log^3 - \log^2$