Toppersnotes
Unleash the topper in you

THE IIT - JEE SECRET

JEE MAINS AND JEE ADVANCED

MATHEMATICS - II

2D & 3D Geometry

SIERRA INNOVATIONS
Private Limited

Contents

Parabola

Ellipse

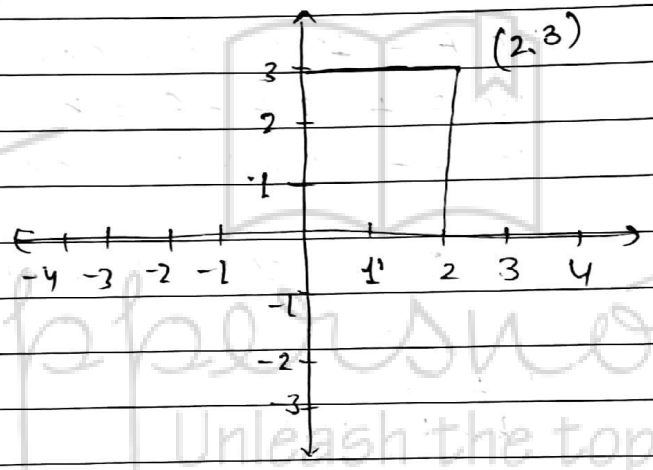
Hyperbola

Toppersnotes
Unleash the topper in you

STRAIGHT LINE

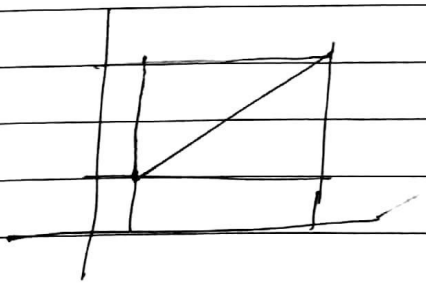
Applications of algebraic techniques in solving geometrical problems is objective of co-ordinate geometry.

Rectangular cartesian Co-ordinate System



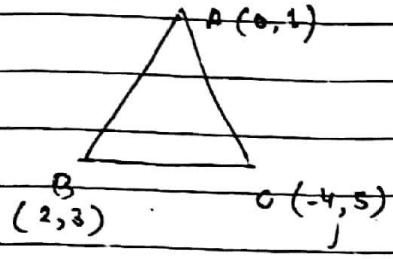
(abscissa) x - co-ord. ⇒ distance from y-axis (with sign)
(ordinate) y - co-ord. ⇒ distance from x-axis (")

Every pt. on x-y plane has unique cartesian co-ordinate



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Points A(0,1), B(2,3) and C(-4,5) forms a triangle find its nature and area of Δ.



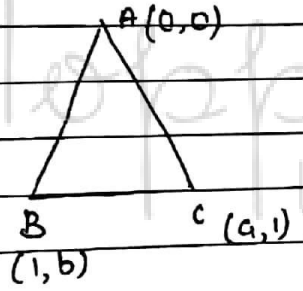
$AB = \sqrt{4+4} = \sqrt{8}$

$AC = \sqrt{16+16} = \sqrt{32}$

$BC = \sqrt{40}$

$\frac{1}{2} \times 4\sqrt{2} \times 2\sqrt{2} = 8$

Q A(0,0), B(1,b) and C(a,1) forms an equilateral Δ then find a, b. (a, b ≥ 0, ab < 1)



$AB^2 = 1 + b^2$

$AC^2 = 1 + a^2$

$BC^2 = (1-a)^2 + (b-1)^2$

$1 + b^2 = 1 + a^2$

$b^2 = a^2 \quad -①$

$1 + b^2 = (1-a)^2 + (b-1)^2$

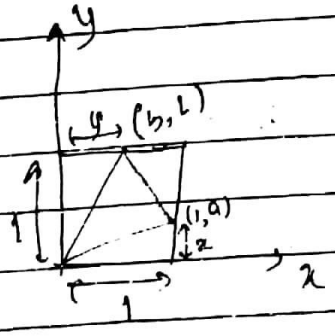
$1 + b^2 = 1 + a^2 - 2a + 1 + b^2 - 2b$

$1 + a^2 = 2a^2 - 4a + 2$

$a^2 - 4a + 1 = 0$

$a = 2 \pm \sqrt{3}$

$= 2 - \sqrt{3}$

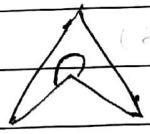


Toppersnotes
Unleash the topper in you

POLYGONS

Convex
All interior angles less than 180°

Concave
Atleast one interior angle greater than 180°
eg:-

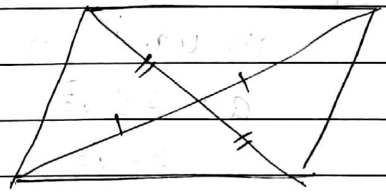


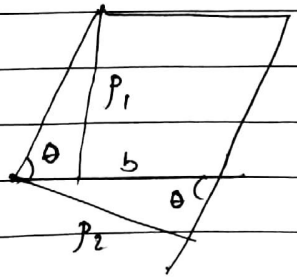
Special Quad.

- Parallelogram
  - Rect.
  - Rhombus
  - Square
- Trapezium
- Kite
- Cyclic quad.

Parallelogram

↳ Diagonals of quad. bisects quad. is ||gm





$A = b \times p_1$

$\sin\theta = \frac{p_2}{b}$

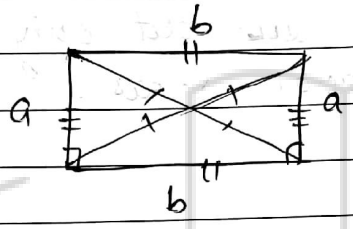
$b = \frac{p_2}{\sin\theta}$

Toppersnotes
Unleash the topper in you

$$A = \frac{p_1 p_2}{\sin\theta}$$

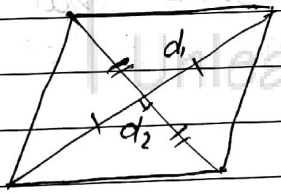
p_1, p_2 are $\perp^r$ dist. b/w pair of $\parallel^l$ sides 'θ' included angle b/w sides

Rectangle



→ Diagonals are equal and bisect eq each other.

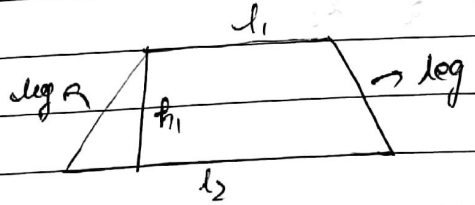
Rhombus



$$A = \frac{1}{2} d_1 d_2$$

→ Diagonals are $\perp^r$ bisector of each other.

Trapezium

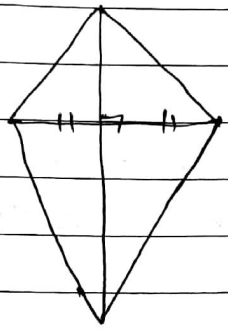


If legs are equal, then it is isosceles trap.

$$A = \frac{1}{2}(l_1 + l_2) h_1$$

Toppersnotes
Unleash the topper in you

Kite

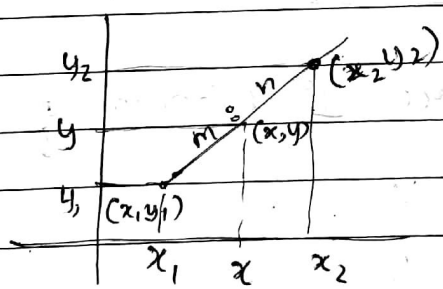


Diagonals are $\perp^{r}$ ars but only one divides it into 2 equal areas.

Section formula

(a) Section formula for internal division

If pt. A, B, C are such that A − C − B, then pts. are collinear and 'C' divides AB internally.



$$\frac{x - x_1}{x_2 - x} = \frac{mx}{nx}$$

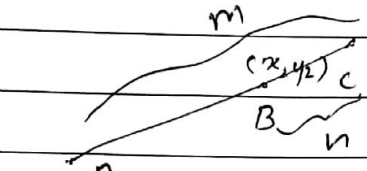
$$nx - nx_1 = mx_2 - mx$$

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

Considers pts. A, B and C such that A − B − C or C − A − B then pts. 'C' divides AB externally

'C' divides AB externally in ratio m:n

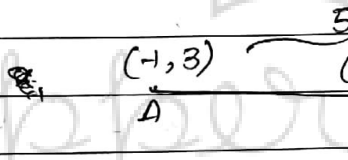


$$\frac{AC}{CB} = \frac{m}{n}$$

$$x = \frac{mx_2 - nx_1}{m-n}$$

$$y = \frac{my_2 - ny_1}{m-n}$$

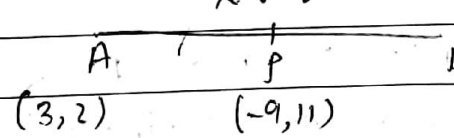
A = (-1, 3), B = (2, 5). Point C divides AB ext. in ratio 5:2. Find co-ordinates of C



$$x = \frac{5\times 2 - 2\times -1}{5-2} = \frac{12}{3} = 4$$

$$y = \frac{25-6}{5-2} = \frac{21}{3} = 7$$

Point P(-9, 11) divides the join of A(3, 2) & B(-1, 5). Find whether the division is internal or external and ratio of division.



$$-9 = \frac{-\lambda + 3}{\lambda + 1} \Rightarrow -9\lambda - 9 = -\lambda + 3$$

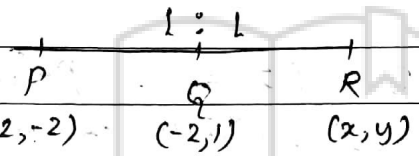
$$8\lambda = -12$$

$$\lambda = -\frac{3}{2}$$

P divides AB ext. in ratio 3:2

$$\frac{AP}{PB} = \frac{3}{2}$$

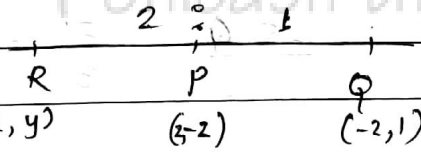
Points P(2,-2) Q(-2,1) and R are collinear such that PR = 10. Find possible co-ordinates of R.



$$PQ = \sqrt{16+9} = 5$$

$$-2+y = 2 \qquad x+2 = -4$$

$$y = 4 \qquad x = -6$$



$$2 = \frac{-4+x}{3} \qquad \frac{2+y}{3} = -2$$

$$x = 10 \qquad y = -8$$

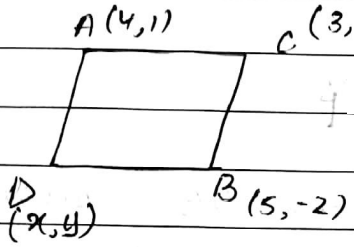
A(4,1), B(5,-2), C(3,7) and D forms a ||gm find the possible co-ordinates of D.

A ≡ (4,1)

B ≡ (5,-2)

C ≡ (3,7)

Toppersnotes
Unleash the topper in you

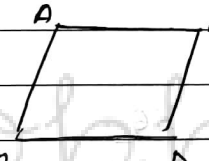


$$\frac{9}{2} = \frac{x+3}{2} \qquad -\frac{1}{2} = \frac{y+7}{2}$$

$$x = 6 \qquad y = -8$$

$$\frac{7}{2} = \frac{x+5}{2} \qquad \frac{8}{2} = \frac{y-2}{2}$$

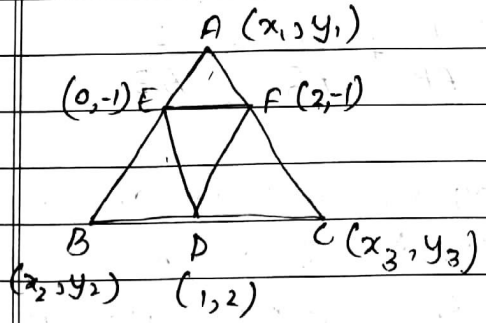
$$x = 2 \qquad y = 10$$



$$\frac{8}{2} = \frac{x+4}{2} \qquad \frac{y+1}{2} = \frac{5}{2}$$

$$x = 4 \qquad y = 4$$

Q. The mid points of the sides of Δs are $(1,2)$ $(0,-1)$ $(2,-1)$. Find the co-ordinates of its vertices.

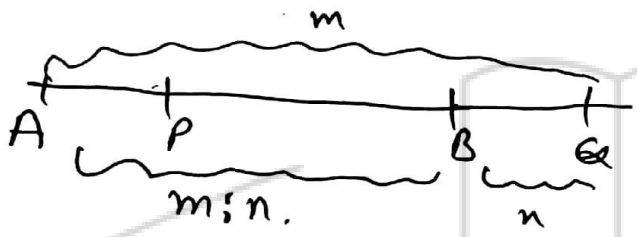


$$x_1 + x_2 = 0$$
$$x_2 + x_3 = 2$$
$$x_3 + x_1 = 4$$
$$x_1 + x_2 + x_3 = 3$$
$$x_1 = 1 \qquad y_3 = 2$$
$$x_2 = -1 \qquad y_2 = 2$$
$$x_3 = 3 \qquad y_1 = -4$$
$$y_1 + y_2 = -2$$
$$y_2 + y_3 = 4$$
$$y_3 + y_1 = -2$$
$$y_1 + y_2 + y_3 = 0$$

Toppersnotes
Unleash the topper in you

Harmonic Conjugate

If Point P divides Segment AB internally in ratio m:n & Q divides it int the ratio m:n externally, then point P & Q are called harmanic Conjugate of each other. with respect AB.



$$\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$$

$$\frac{AP}{AB} = \frac{AQ}{QB}$$

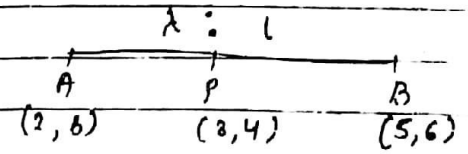
$$\frac{AP}{AB-AP} = \frac{AQ}{AQ-AB}$$

$$\frac{AB}{AP} - 1 = 1 - \frac{AB}{AQ}$$

$$AB\left(\frac{1}{AP} + \frac{1}{AQ}\right) = 2$$

$$\frac{1}{AP} + \frac{1}{AQ} = \frac{2}{AB}$$

The line segment joining A(2,3) and B(5,6) is divided by P(3,4) find harmonic conjugate of P with respect to AB.



$$3 = \frac{5\lambda + 2}{\lambda + 1}$$

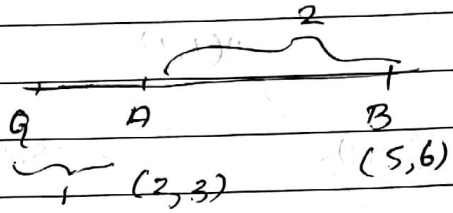
Toppersnotes
Unleash the topper in you

$3\lambda + 3 = 5\lambda + 2$

$\lambda = 1/2$

$\frac{AP}{BP} = \frac{1}{2}$

$\frac{AQ}{BQ} = \frac{1}{2}$

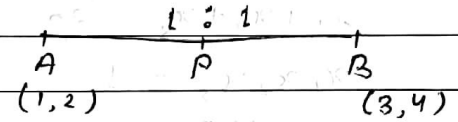


$x = \frac{4-5}{2-1}$ $\quad y = \frac{6-6}{2-1}$

$x = -1$ $\quad y = 0$

$Q \equiv (-1, 0)$

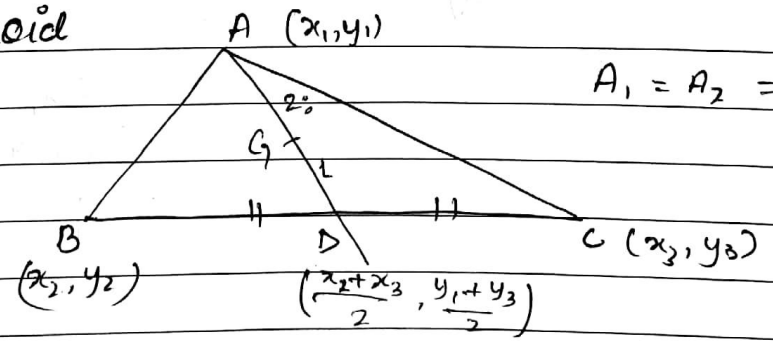
A (1, 2) and B (3, 4). Find harmonic conjugate if P is mid point.



There will be no external division.

Some Special points of Δ.

Centroid



$A_1 = A_2 = A_3 = A/3$

$x_g = \frac{x_2 + x_3 + x_1}{3}$ $\quad y_g = \frac{y_2 + y_3 + y_1}{3}$