



THE IIT - JEE SECRET

JEE MAINS AND JEE ADVANCED

MATHEMATICS - II
2D & 3D Geometry



Contents

1. Straight Line	01-88
a. Cartesian system and distance formula	1
b. Section formula	5
c. Harmonic conjugate	9
d. Special points of triangle	10
e. Area of triangle	18
f. Area of quadrilateral	19
g. Locus	21
h. Slope of Straight line	27
i. Angle between the lines	28
j. X and Y intercept of the line	30
k. Different forms of the line	31
l. Position of a point w.r.t. a line	46
m. Length of perpendicular to a line	47
n. Image of a point in a line	55
o. Condition of concurrency of lines	59
p. Family of lines	61
q. Shifting of origin	71
r. Rotation of coordinate axis	72
s. Pair of straight lines	73
t. General second degree equation in X and Y	79
u. Homogenization	80
v. Angle bisector	82
2. Circle	89-136
a. Definition and general equation of circle	89
b. Nature of circle	89

c. Condition of second degree equation to represent a circle	90
d. Diametric form of circle	92
e. Intercepts of circle	94
f. Position of a point w.r.t. to a circle	98
g. Parametric equation of a circle	99
h. Line and circle	101
i. Tangent to a circle	102
j. Different forms of tangent to the circle	103
k. Point of tangency	104
l. Length of tangent	105
m. Power of point	107
n. Chord with a given middle point	108
o. Chord of contact	111
p. Pole and polar	113
q. Conjugate points and lines	114
r. Combined equation of pair of tangents	115
s. Family of circles	115
t. Common tangents	121
u. Length of common tangents	122
v. Equation of common tangents	123
w. Radical axis and radical Centre	126
x. Co- axial system of circles	131
y. Orthogonal circles	133
z. Director circle	136

3. Solution of Triangle 137-167

a. Sine law	137
b. Cosine law	140
c. Projection formula	144
d. Napier analogy (Tangent formula)	145
e. Half angle formula	148
f. M-n theorem	152
g. Circumcircle	154

h. Incircle	154
i. Ex- circle	157
j. Orthocenter	162
k. Pedal triangle	162
l. Length of angle bisector	167
m. Length of median	167

4. Conic Section 169-249

General introduction	169
-----------------------------	------------

Parabola

a. Terminology	171
b. Parametric coordinates	177
c. Tangent to the parabola	180
d. Slope form of the tangent	
	181
e. Director circle	183
f. Position of a point w.r.t. parabola	187
g. Normal to parabola from a given point	188
h. Condition of 3 normals from a point	189
i. Concept of envelope	192
j. Chord of contact	193
k. Combined equation of pair of tangent	193
l. Highlights	198

Ellipse

a. Terminology	203
b. Definition of ellipse	204
c. Position of a point w.r.t. ellipse	209
d. Auxiliary circle , Eccentric angle , parametric coordinates	210
e. Tangent to ellipse	211
f. Director circle	214
g. Normal to ellipse	216
h. Highlights	217
i. Reflection property	219

Hyperbola

a. Terminology	224
b. Alternate definition of hyperbola	225
c. Conjugate hyperbola	228
d. Position of a point w.r.t. hyperbola	229
e. Auxiliary circle	230
f. Tangent to the hyperbola	231
g. Normal to the hyperbola	236
h. Chord of contact / chord with a given middle point / pair of tangents	237
i. Highlights	240
j. Asymptotes	243
k. Highlights of asymptotes	245
l. Rectangular Hyperbola	247

5. Vector **250-307**

a. Scalar and vector quantities	250
b. Mathematical description of vectors	250
c. Types of vector	250
d. Scalar multiple of a vector	252
e. Collinear vectors	252
f. Co-planarity of vectors	254
g. Vector law of addition (Triangle law, Parallelogram law)	254
h. Position vector	256
i. Section formula for position vector	257
j. Tetrahedron	259
k. Parallelepiped	259
l. Vector equation of line	260
m. Equation of angle bisector	265
n. Dot product	267
o. Linear combination of vectors	275

p. Fundamental theorem in plane	275
q. Cross product or vector product	278
r. Area vectors	281
s. Lagrange identity	284
t. Shortest distance between two skew lines	286
u. Distance between parallel lines	288
v. Volume of tetrahedron	290
w. Vector triple product	295
x. Scalar product of 4 vectors	297
y. Vector product of 4 vectors	299
z. Fundamental theorem in space	300
aa. Linearly independent system of vectors	303
bb. Reciprocal system of vectors	306

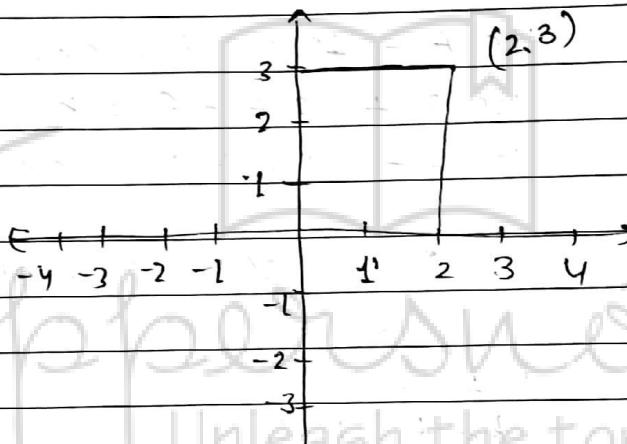
6. 3D Geometry 308-331

a. Direction cosine and direction angle of vector	308
b. Direction ratio of the line	310
c. Plane	313
d. Equation of reference plane	313
e. Vector equation of plane	314
f. Intercept form of plane	317
g. Normal form of plane	317
h. Parametric form of plane	318
i. Condition of coplannarity of lines	319
j. Bisector planes	322
k. Family of plane	322
l. Angle between line and plane	324
m. symmetrical form of line	325
n. Miscellaneous problems	326

STRAIGHT LINE

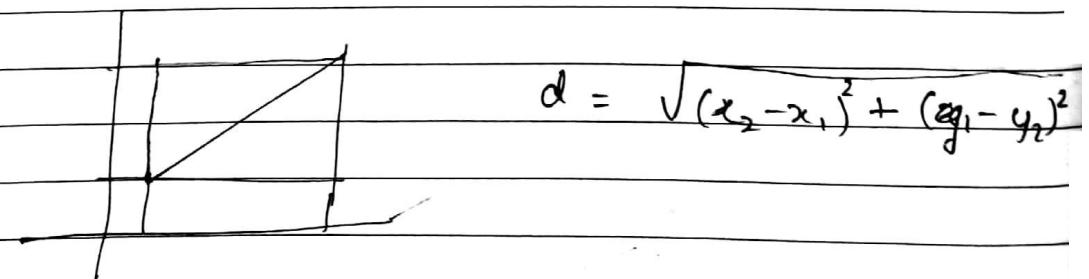
Applications of algebraic techniques in solving geometrical problems is objective of co-ordinate geometry.

Rectangular cartesian Co-ordinate System

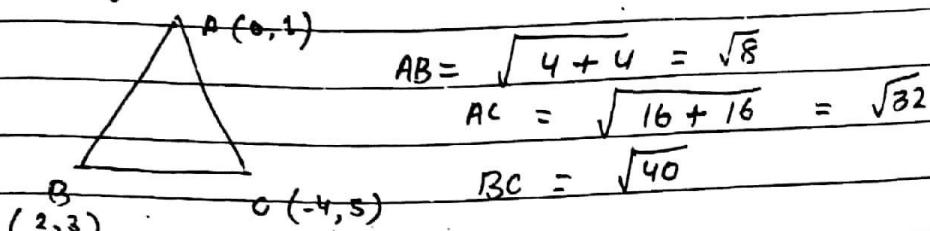


(abscissa) x - co-ord. \Rightarrow distance from y -axis (with sign)
 (ordinate) y - co-ord. \Rightarrow distance from x -axis (\gg)

Every pt. on $x-y$ plane has unique cartesian co-ordinates



Points $A(0, 1)$, $B(2, 3)$ and $C(-4, 5)$ forms a triangle find its nature and area of Δ .



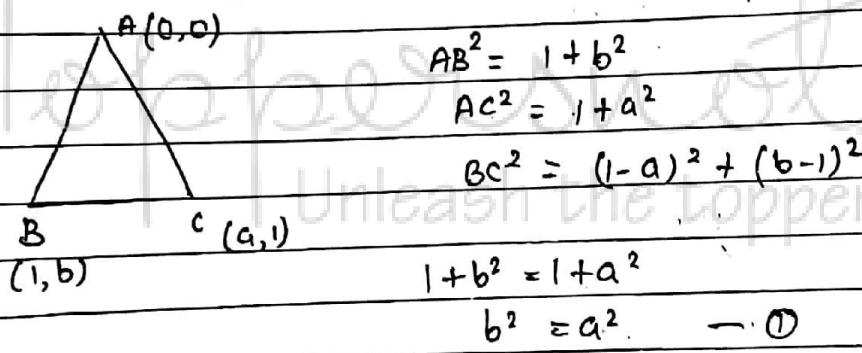
$$AB = \sqrt{4+4} = \sqrt{8}$$

$$AC = \sqrt{16+16} = \sqrt{32}$$

$$BC = \sqrt{40}$$

$$\frac{1}{2} \times 4\sqrt{2} \times 2\sqrt{2} = 8$$

If $A(0, 0)$, $B(1, b)$ and $C(a, 1)$ forms an equilateral Δ then find a, b . ($a, b \geq 0, b < 1$)



$$AB^2 = 1 + b^2$$

$$AC^2 = 1 + a^2$$

$$BC^2 = (1-a)^2 + (b-1)^2$$

$$1 + b^2 = 1 + a^2$$

$$b^2 = a^2 \quad \dots \textcircled{1}$$

$$1 + b^2 = (1-a)^2 + (b-1)^2$$

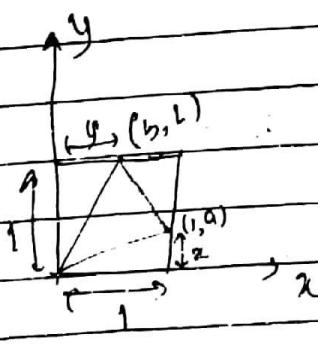
$$1 + b^2 = 1 + a^2 - 2a + 1 + b^2 - 2b$$

$$1 + a^2 = 2a^2 - 4a + 2$$

$$a^2 - 4a + 1 = 0$$

$$a = 2 \pm \sqrt{3}$$

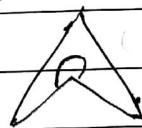
$$= 2 - \sqrt{3}$$



POLYGONS

Convex
all interior
angles less than
 180°

Concave
at least one interior
angle greater than 180°
eg:-



Special Quad.

Rect.

Parallelogram

Rhombus

Square

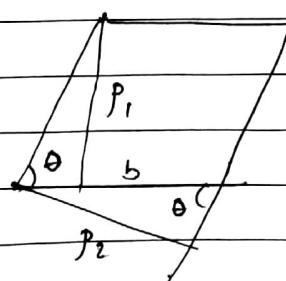
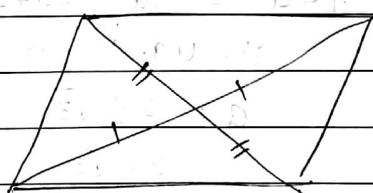
Trapezium

Kite

Cyclic quad.

Parallelogram

→ Diagonals of quad. bisects quad. is \parallel



$$A = b \times p_1$$

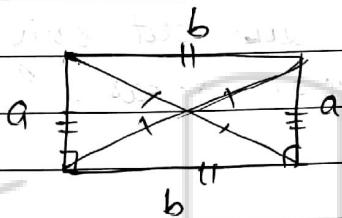
$$\sin \theta = \frac{p_2}{b}$$

$$b = \frac{p_2}{\sin \theta}$$

$$A = \frac{P_1 P_2}{\sin \theta}$$

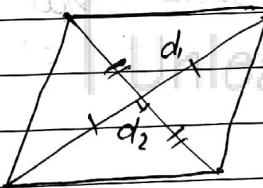
P_1, P_2 are \perp^{st} dist. b/w pair of 11^{th} sides θ'
included angle b/w sides

Rectangle



→ Diagonals are equal and bisect each other.

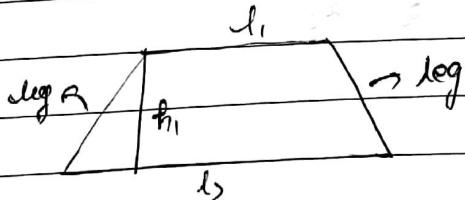
Rhombus



$$A = \frac{1}{2} d_1 d_2$$

→ Diagonals are \perp^{st} bisectors of each other.

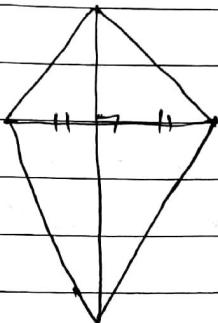
Trapezium



If legs are equal, then it is isosceles trap.

$$A = \frac{1}{2} (l_1 + l_2) h$$

Kite

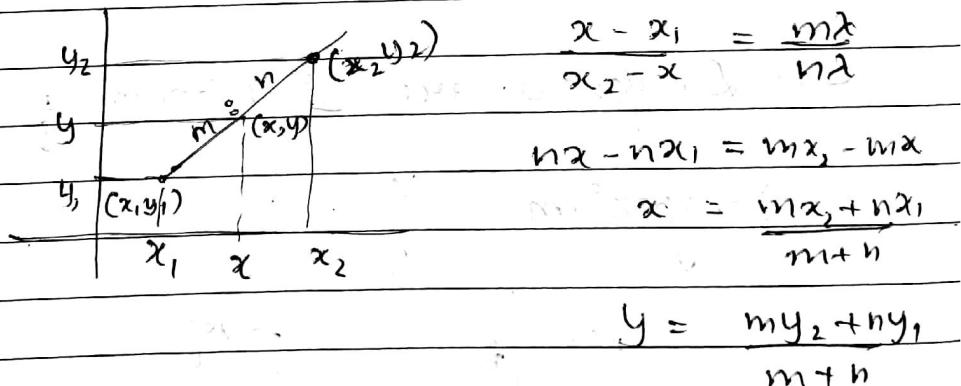


Diagonals are \perp^{r} but only one divides it into 2 equal areas.

Section formula

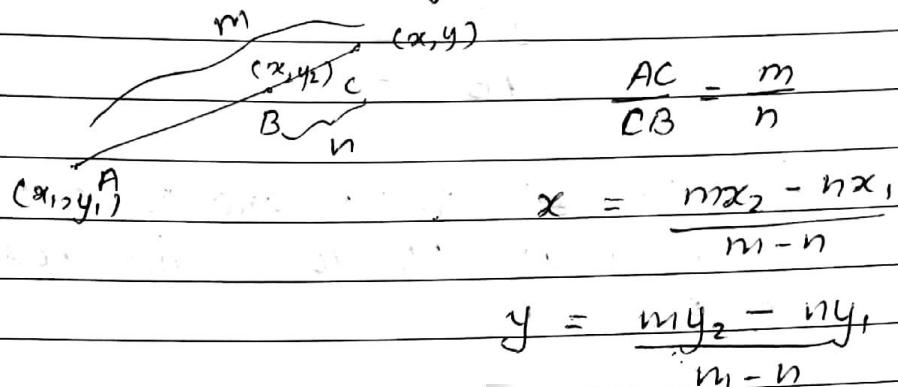
(a) Section formula for internal division

If pts. A, B, C are such that A - C - B, then pts. are collinear and 'c' divides AB internally.

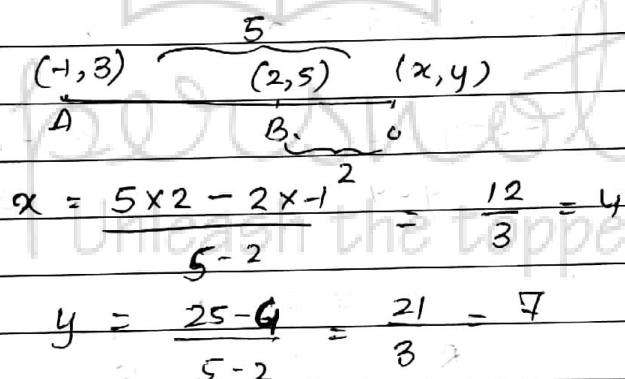


Consider pts. A, B and c such that A-B-C or C-A-B then pts. 'c' divides AB externally

'C' divides AB externally in ratio m:n



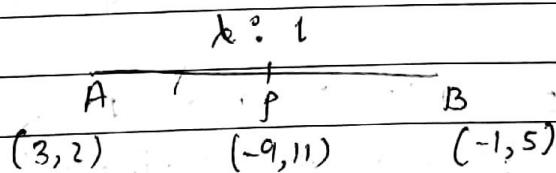
~~A = (-1, 3), B = (2, 5). Point c divides AB, in ratio 5:2. Find co-ordinates of C~~



$$x = \frac{5 \times 2 - 2 \times (-1)}{5-2} = \frac{12}{3} = 4$$

$$y = \frac{25 - 9}{5-2} = \frac{21}{3} = 7$$

~~Point P (-9, 11) divides the join of A (3, 2) & B (-1, 5). Find whether the division is internal or external and ratio of division.~~



$$-9 = \frac{-\lambda + 3}{\lambda + 1} \Rightarrow -9\lambda - 9 = -\lambda + 3$$

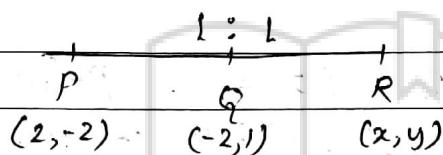
$$8\lambda = -12$$

$$\lambda = -\frac{3}{2}$$

P divides AB ext. in ratio 3:2

$$\frac{AP}{PB} = \frac{3}{2}$$

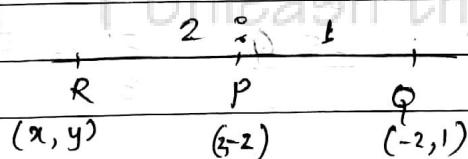
Points P(2, -2), Q(-2, 1) and R are collinear such that PR = 10. Find possible co-ordinates of R.



$$PQ = \sqrt{16+9} = 5$$

$$\begin{aligned} -2+y &= 2 \\ y &= 4 \end{aligned}$$

$$\begin{aligned} x+2 &= -4 \\ x &= -6 \end{aligned}$$



$$2 = \frac{-4+x}{3}$$

$$\frac{2+y}{3} = -2$$

$$x = 10$$

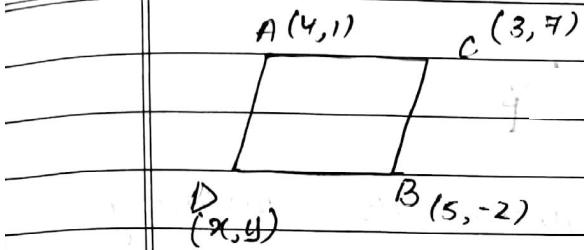
$$y = -8$$

A(4, 1), B(5, -2), C(3, 7) and D forms a ||gm. find the possible co-ordinates of D.

$$A \equiv (4, 1)$$

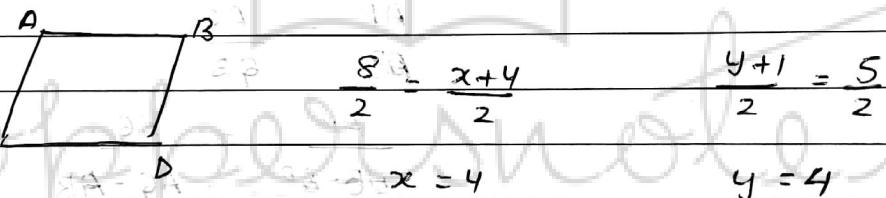
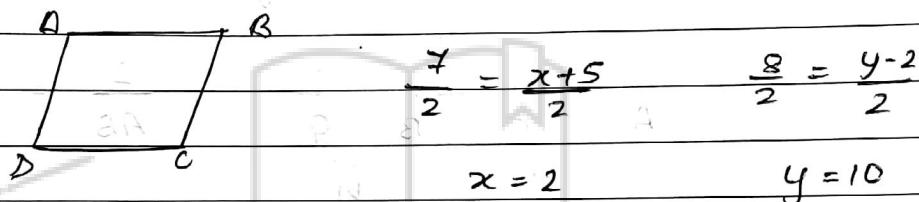
$$B \equiv (5, -2)$$

$$C \equiv (3, 7)$$



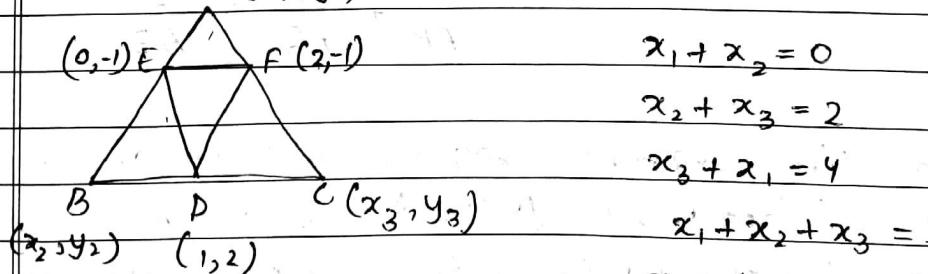
$$\frac{9}{2} = \frac{x+3}{2} + \frac{-1-7}{2} = \frac{y+7}{2}$$

$$x = 6 \quad y = -8$$



Q. The mid points of the sides of \triangle s are $(1, 2)$, $(0, -1)$, $(2, -1)$. Find the co-ordinates of its vertices.

A (x_1, y_1)



$$x_1 = 1 \quad y_3 = 2$$

$$x_2 = -1 \quad y_2 = 2$$

$$x_3 = 3 \quad y_1 = -4$$

$$y_1 + y_2 = -2$$

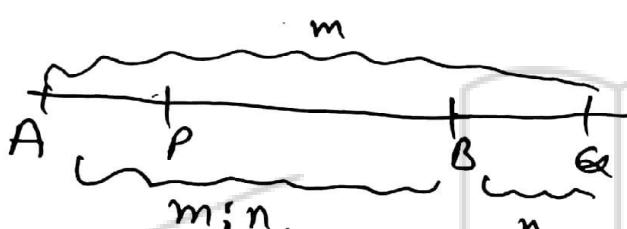
$$y_2 + y_3 = 4$$

$$y_3 + y_1 = -2$$

$$y_1 + y_2 + y_3 = 0$$

Harmonic Conjugate

If point P divides segment AB internally in ratio $m:n$ & Q divides it in the ratio $m:n$ externally, then points P & Q are called harmonic conjugates of each other with respect to AB.



$$\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$$

$$\frac{AP}{AB} = \frac{AQ}{QB}$$

$$\frac{AP}{AB - AP} = \frac{AQ}{AQ - AB}$$

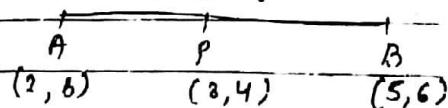
$$\frac{AB}{AP} - 1 = 1 - \frac{AB}{AQ}$$

$$AB \left(\frac{1}{AP} + \frac{1}{AQ} \right) = 2$$

$$\frac{1}{AP} + \frac{1}{AQ} = \frac{2}{AB}$$

The line segment joining A(2, 3) and B(5, 6) is divided by P(3, 4) find harmonic conjugate of P with respect to AB.

$$\lambda : 1$$



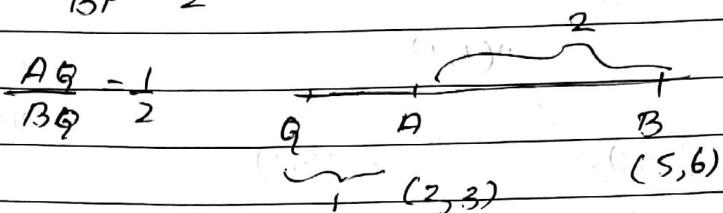
$$3 = \frac{5\lambda + 2}{\lambda + 1}$$

$$3\lambda + 3 = 5\lambda + 2$$

$$\lambda = \frac{1}{2}$$

$$\frac{AP}{BP} = \frac{1}{2}$$

$$\frac{AQ}{BQ} = \frac{1}{2}$$

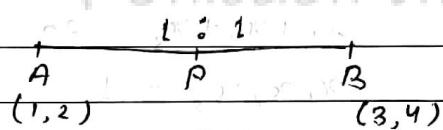


$$x = \frac{y-5}{2-1} \quad y = \frac{6-6}{2-1}$$

$$x = -1 \quad y = 0$$

$$Q \in (-1, 0)$$

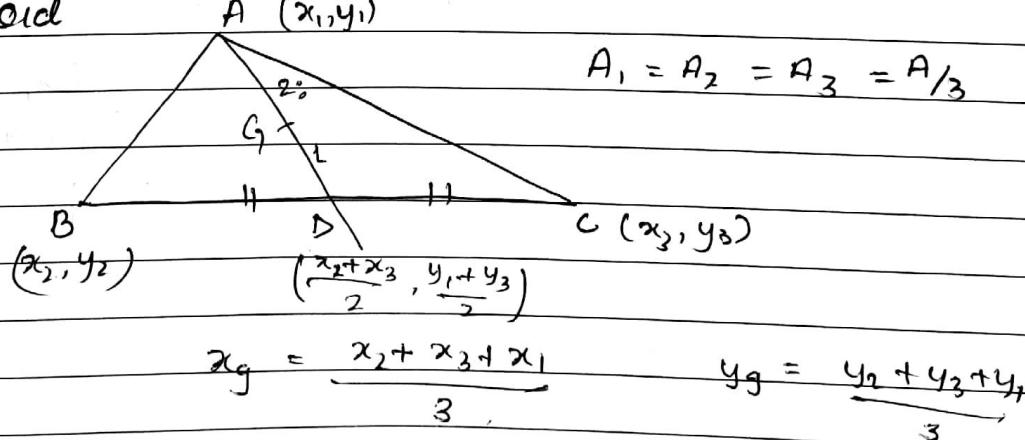
A(1, 2) and B(3, 4). Find harmonic conjugate if P is mid point.



There will be no external division.

Some Special points of Δ .

Centroid



$$A_1 = A_2 = A_3 = A/3$$