Toppersnotes
Unleash the topper in you

THE IIT - JEE SECRET

JEE MAINS AND JEE ADVANCED

MATHEMATICS - IV

Calculus - II

SIERRA INNOVATIONS
Private Limited

Contents

INDEFINITE INTEGRATION

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad , n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + c \qquad \left[\frac{d}{dx} \ln|x| = \frac{1}{x} \right]$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int e^x dx = e^x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x = \cot x + c$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$$

$$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + c$$

$$\int \frac{dx}{x^2+1} = \tan^{-1} x + c$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int k f(x) dx = k \int f(x) dx.$$

$$\int f(x) . dx = g(x) + c$$

$$\int f(ax+b) dx = \frac{g(ax+b)}{a} + c$$

$$\int \cos(2x+3) = \frac{\sin(2x+3)}{2} + c$$

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c$$

$$\int \frac{dx}{\sqrt{ax+b}} = (ax+b)^{-1/2} \, dx = \frac{2(ax+b)^{1/2}}{(a)}$$

$$\int \frac{dx}{(ax+b)^2} = \int (ax+b)^{-2} dx = \frac{(ax+b)^{-1}}{-a}$$

$$\int e^{\ln\sqrt{x}} dx.$$

$$e^{\ln\sqrt{x}}$$

$$\frac{2x^{3/2}}{3} + c$$

$$\int \frac{\sqrt{x^4 - x^{-4} + 2}}{x^3} \, dx$$

$$\left(\frac{x^4 - x^{-4} + 2}{x^3}\right)^{1/2} dx$$

$$(x^4 - x^{-4} + 2)^{1/2} x^{-3} \, dx$$

$$\int \frac{x^2 + x^{-2}}{x^3} \, dx.$$

$$\int \frac{1}{x} + x^{-5} \, dx.$$

$$\ln|x| + \frac{x^{-4}}{-4} + c$$

$$\int \frac{x\sqrt{x} - 1}{x + \sqrt{x} + 1} \, dx$$

$$t = \sqrt{x}.$$

$$\int \frac{t^3 - 1}{t^2 + t + 1} \, dx$$

$$\int (\sqrt{x} - 1) \, dx$$

$$\frac{2x^{3/2}}{3} - x + c$$

$$\int \frac{dx}{ax+b} = \frac{\ln|ax+b|}{a} + c$$

$$\int \frac{dx}{a} = \lambda \ln|\ \ | + k$$

$$\int \frac{dx}{x^2+3x-10} = \frac{1}{7}\int \frac{(x+5)-(x-2)}{(x+5)(x-2)}$$

$$= \frac{1}{7}\ln\left|\frac{x-2}{x+5}\right| + c$$

$$\int \frac{dx}{3-2x} = \frac{\ln(3-2x)}{-2} + c$$

$$\int \frac{dx}{2x^2-x-1} = \int \frac{dx}{2x^2-2x+x-1} = \int \frac{dx}{2x(x-1)+(x-1)}$$

$$\frac{1}{3}\int \frac{(2x+1)-2(x-1)}{(x-1)(2x+1)}dx = \frac{1}{3}\int \frac{1}{x-1} - 2\cdot\frac{1}{2x+1}\,dx$$

$$= \frac{1}{3}\left[\ln|x-1| - 2\,\frac{\ln(2x+1)}{2}\right]$$

$$\int \frac{5x+9}{x^2+3x-10}\,dx = \int \frac{5x+9}{(x-2)(x+5)}\,dx = \int \frac{3(x+5)-2(x-2)-2}{(x+5)(x-2)}$$

$$\int \frac{3}{x-2} + \frac{2}{x+5} - \frac{2}{(x+5)(x-2)}\,dx = 3\ln|x-2| + 2\ln|x+5| - \frac{1}{7}\ln\left|\frac{x-2}{x+5}\right|$$

$$\int \frac{x}{x+1}\,dx$$

$$\int \frac{x+1-1}{x+1}\,dx$$

$$x - \int \frac{1}{x+1}\,dx$$

$$x - \ln|x+1|$$

$$\int \frac{2x-3}{3x-2}\,dx$$

$$= \frac{2}{3}\int \frac{6x-9}{6x-4}\,dx$$

$$\frac{2}{3}\int \frac{6x-4-5}{6x-4}\,dx$$

$$\frac{2}{3}\left[x - \frac{5}{6}\ln|6x-4| + c\right]$$

$$\int a^{px+q}\,dx = \frac{a^{px+q}}{p(\ln a)} + c$$

$$\int 3^{5x-4}\,dx = \frac{3^{5x-4}}{(\ln 3)(5)} - C$$

$$\int 3^{x}5^{-x}.dx = \int \left(\frac{3}{5}\right)^{x}.dx = \frac{\left(\frac{3}{5}\right)^{x}}{\ln\left(\frac{3}{5}\right)}$$

$$\int \frac{a^x+b^x+c^x}{(abc)^x}.dx.$$

$$\int \left(\frac{1}{bc}\right)^{x} + \left(\frac{1}{ac}\right)^{x} + \left(\frac{1}{ab}\right)^{x} dx$$

$$\frac{\left(\frac{1}{bc}\right)^{x}}{\ln\left(\frac{1}{bc}\right)} + \frac{\left(\frac{1}{ac}\right)^{x}}{\ln\left(\frac{1}{ac}\right)} + \frac{\left(\frac{1}{ab}\right)^{x}}{\ln\left(\frac{1}{ab}\right)}$$

$$\int \frac{e^{3x}+e^{5x}}{e^{x}+e^{-x}}.dx$$

$$\int \frac{e^{4x}+e^{6x}}{e^{2x}+1}.dx$$

$$\int e^{4x}.dx$$

$$= \frac{e^{4x}}{4} + c$$

$$\int a^{mx}.b^{nx}.dx$$

$$\int (a^m.b^n)^x dx$$

$$= \frac{a^m.b^n}{\ln(a^m.b^n)} + c$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos^2\theta = 1+\cos 2\theta.$$

$$2\sin^2\theta = 1-\cos 2\theta$$

$$\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\sin^2 2x$$

$$\sin^6 x + \cos^6 x = 1 - \frac{3}{4}\sin^2 2x$$

$\int \cos(5-4x)\,dx = \frac{\sin(5-4x)}{-4} + c$

$\int \cos 2x \cos 3x\,dx = \frac{1}{2}\int (\cos(5x) + \cos x)\,dx$

$= \frac{1}{2}\left[\frac{\sin 5x}{5} + \sin x\right] + c$

$\int \cos^2 3x\,dx$

$\int (1+\cos 6x)\,dx$

$x + \frac{\sin 6x}{6} + c$

$\int (\cos^4 x - \sin^4 x)\,dx$

$= \int \left(1 - \frac{1}{2}\sin^2 2x\right) dx$

$= x - \frac{1}{2}\int \sin^2 2x\,dx$

$= x - \frac{1}{2}\int (1-\cos 4x)\,dx$

$= x - \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right] + c$

$= \int \cos^2 x$

$\int (\sin^6 x + \cos^6 x)\,dx$

$\int \cos^4 x\,dx.$

$= \int (\cos^2 x)^2\,dx$

$= \int (1+\cos 2x)^2\,dx$

$\int \frac{(1+\cos 2x)^2}{4}\,dx$

$= \frac{1}{4}\left[x + \sin 2x + \int \frac{1+\cos 4x}{2}\right.$

$= \frac{x + \sin 2x}{4} + \frac{1}{8}\left(x + \frac{\sin 4x}{4}\right)$

$\int \frac{\cos x - \cos 2x}{1-\cos x}\,dx$

$= \int \frac{2\cos^2 x - \cos x - 1}{\cos x - 1}\,dx$

$= \int \frac{(\cos x - 1)(2\cos x + 1)}{(\cos x - 1)}\,dx$

$= 2\sin x + x + c$

$$\int \sec^2(ax+b)\,dx = \frac{\tan(ax+b)}{a} + c$$

$$\int \operatorname{cosec}^2(ax+b).dx = -\frac{\cot(ax+b)}{a} + c$$

$$\int \sec^2(3-4x)\,dx = \frac{\tan(3-4x)}{-4} + c$$

$$\int \frac{dx}{1+\cos x}$$

$$\int \frac{dx}{2\cos^2\frac{x}{2}} = \frac{1}{2}\int \sec^2\frac{x}{2}\,dx = \frac{2}{2}\tan\frac{x}{2} = \tan\frac{x}{2} + C$$

$$\int \frac{1-\cos 2x}{1+\cos 2x}.dx$$

$$\int \frac{2\sin^2 x}{2\cos^2 x}\,dx$$

$$\int (-1+\sec^2 x)\,dx$$

$$\tan x - x + C$$

$$\int \cot^2 x.\,dx$$

$$\int (\operatorname{cosec}^2 x - 1)\,dx$$

$$= -\cot x - x$$

$$\int \tan^2 x \sin^2 x.dx$$

$$= \int (\sec^2 x - 1)\sin^2 x.dx$$

$$= \int \tan^2 x - \int \sin^2 x.$$

$$= \tan x - x - \frac{1}{2}\int 1-\cos 2x.\,dx$$

$$= \tan x - x - \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]$$

$$\int \sec(ax+b)\tan(ax+b)\,dx = \frac{\sec(ax+b)}{a} + c$$

$$\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = -\frac{\operatorname{cosec}(ax+b)}{a} + c$$

$$\int \frac{dx}{1-\sin 3x}$$

or $\left(\frac{\pi}{2} - 3x\right)$

$$\int \frac{1+\sin 3x}{\cos^2 3x}$$

$$\int \sec^2 3x + \tan 3x \sec 3x$$

$$\frac{\tan 3x}{3} + \frac{\sec 3x}{3} + c$$

$$\int \frac{a\sin^3 x + b\cos^3 x}{\sin^2 x\cos^2 x}\,dx$$

$$a\int \sec x \tan x + b\int \operatorname{cosec} x \cot x$$

$$a\sec x + b\operatorname{cosec} x + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$$

$$= \frac{1}{a}\cdot\frac{1}{\frac{x}{a}\sqrt{\frac{x^2}{a^2}-1}}\cdot\frac{1}{a}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{25-(2x-3)^2}} = \frac{1}{2}\sin^{-1}\left(\frac{2x-3}{5}\right) + c$$

$$\int \frac{dx}{|2x-1|\sqrt{4x^2-4x}} \quad \int \frac{dx}{|2x-1|\sqrt{(2x-1)^2-1}} \quad \frac{1}{2}\sec^{-1}(2x-1) + c$$

Differentials

$$y = f(x)$$

$$dy = f'(x)\,dx$$

$$\tan\theta = \frac{dy}{dx}$$

$dx \cdot f'(x) = dy$

$d(\sin x) = \cos x \, dx$

$d(\ln x) = \frac{1}{x} \cdot dx$

Integration by substitution

$\int e^x \sin(e^x)\, dx$

$e^x = t$

$e^x \cdot dx = dt$

$\int \sin t \cdot dt$

$= c - \cos t$

$= c - \cos(e^x)$

$\int \cos x^2 \; x \cdot dx$

$x^2 = t$

$2x \cdot dx = dt$

$\int \frac{1}{2} \cos t \, dt$

$\frac{1}{2} \sin x^2 + C$

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \cdot dx$

$\sqrt{x} = t$

$\frac{1}{\sqrt{x}} dx = dt$

$= \int e^t \cdot dt$

$= 2e^{\sqrt{x}}$

$\int \cos(\sin x) \cos x \cdot dx$

$\sin x = t$

$\cos x \, dx = dt$

$\int \cos t \, dt$

$\sin(\sin x) + C$

$\int \frac{(\tan^{-1} x)^2}{1+x^2} dx$

$\tan^{-1} x = t$

$\frac{1}{1+x^2} dx = dt$

$\int t^2 \cdot dt$

$\frac{t^3}{3}$

$\frac{(\tan^{-1} x)^3}{3} + C$

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$$\int \frac{\cos(\sin^{-1}x)}{\sqrt{1-x^2}}\,dx$$

$\sin^{-1}x = t$

$\frac{1}{\sqrt{1-x^2}}\,dx = dt$

$\int \cos(t)\,dt$

$\sin(\sin^{-1}x) + C$

$$\int \frac{\tan\sqrt{x}\,\sec^2\sqrt{x}}{\sqrt{x}}\,dx$$

$\tan\sqrt{x} = t$

$\frac{\sec^2\sqrt{x}}{2\sqrt{x}}\,dx = dt$

$\int 2t\,dt$

t^2

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$f(x) = t$

$f'(x)\,d(x) = dt$

$\int \cot x\,dx = \ln|\sin x| + c$

$\int \tan x\,dx = \ln|\sec x| + c$

$\int \sec x\,dx = \ln|\sec x + \tan x| + c$

$\frac{s(s+T)}{(s+T)} = \frac{sT+s^2}{s+T} = \ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c$

$\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + c$

$= \ln\left|\tan\frac{x}{2}\right| + c$

$$\int \frac{\cos x - \sin x}{\sin x + \cos x}\,dx$$

$\int \frac{\cos 2x}{1+\sin 2x}\,dx = \int \frac{1-\tan x}{1+\tan x}\,dx$

$\ln|\sin x + \cos x| + C$

$$\int \frac{e^x}{e^x+1}\,dx$$

$$\int \frac{e^x+1-1}{e^x+1}\,dx$$

$$\int 1-\frac{1}{e^x+1}\,dx$$

$$x-\int\frac{1}{e^x+1}\,dx = \ln|e^x+1|+c$$

$$\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$$

$$\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$$

$$\ln(e^x+e^{-x})+c$$

$$\int \frac{1}{e^x+1}\,dx$$

$$-\int\frac{-e^{-x}}{1+e^{-x}}\,dx$$

$$= -\ln(1+e^{-x})+c$$

$$= -\ln\left(\frac{e^x+1}{e^x}\right)+c$$

$$= \{\ln(e^x+1)-\ln e^x\}+c$$

$$\int \frac{f'(x)}{\sqrt{f(x)}}\,dx$$

$$= 2\sqrt{f(x)}+c$$

$$\sqrt{f(x)}=t$$

$$\int \frac{\sec^4 x}{\sqrt{\tan x}}\,dx$$

$$\int \frac{1+\tan^2 x}{\sqrt{\tan x}}\sec^2 x\,dx$$

$$\tan x = t$$

$$\int \frac{1+t^2}{t^{1/2}}$$

$$2\sqrt{t}+\frac{2}{5}t^{5/2}+c$$

$$\frac{\sin(A-B)}{\cos A\cos B} = \tan A + \tan B$$

$$\frac{\sin(A-B)}{\sin A\sin B} = \cot B - \cot A$$

$$\int \frac{\sin x}{\sin 2x \sin 3x} dx.$$

$$\int \frac{\sin(3x-2x)}{\sin^2 x \sin 3x} \cdot dx$$

$$-\int \cot 3x + \int \cot 2x \cdot dx$$

$$\frac{\ln(\sin 2x)}{2} - \frac{\ln|\sin 3x|}{3} + c$$

$$\int \frac{dx}{\cos(x-a)\cos(x-b)}$$

$$\int \frac{\sin((x-a)-(x-b))}{\cos(x-a)\cos(x-b)}$$

$$\frac{1}{\sin(b-a)} \int \tan(x-a) + \tan(x-b)$$

b-a

$$\int \sin^2 x \cos^3 x \, dx$$

$$\int \sin^2 x \cos^2 x \cos x \, dx$$

$$\int \sin^2 x (1 - \sin^2 x) \cos x \, dx$$

$\sin x = t$

$$\int (t^2 - t^4) dt$$

$$\int \frac{\sin 2x}{b\cos^2 x + a\sin^2 x} dx$$

$b\cos^2 x + a\sin^2 x = t$

$(a-b)\sin 2x \, dx = dt$

$$\frac{1}{a-b} \int \frac{dt}{t^{100}}$$

$$\int (\tan^3 x - x\tan^2 x) dx$$

$$\int (\tan x - x)(\tan^2 x) dx$$

$\tan x - x = t$

$$\int t \, dt$$

$$\int \frac{x\cos x \, dx}{(x\sin x + \cos x)^2}$$

Standard substitution

$\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$

$\sqrt{x^2+a^2}$ $\quad x = a\tan\theta$

$\sqrt{x^2-a^2}$ $\quad x = a\sec\theta$

$\sqrt{\frac{a-x}{a+x}}$ $\quad x = a\cos\theta$

$$\int \frac{x^2}{\sqrt{a^6-x^6}}\,dx.$$

$x^3 = a^3\sin\theta$

$3x^2dx = a^3\cos\theta\,d\theta$

$\theta = \sin^{-1}\frac{x^3}{a^3}$

$$\frac{1}{3}\int \frac{a^3\cos\theta\,d\theta}{a^3\cos\theta} = \frac{1}{3}\theta + c$$

$$\frac{1}{3}\sin^{-1}\frac{x^3}{a^3} + c$$

$$\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}\,dx$$

$$\int \frac{x^{1/2}}{\sqrt{(a^{3/2})^2-(x^{3/2})^2}}$$

$x^{3/2} = a^{3/2}\sin\theta$

same as above

$$\int \frac{\sqrt{(9-x^2)^3}}{x^6}.dx$$

$\cosec\theta$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$$\int \frac{3^4}{3^6}\cdot\frac{\cos^4\theta}{\sin^6\theta}\,d\theta$$