



THE IIT - JEE SECRET

JEE MAINS AND JEE ADVANCED

MATHEMATICS - IV
Calculus - II



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INDEFINITE INTEGRATION

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C \quad \left[\frac{d}{dx} (\ln|x|) = \frac{1}{x} \right]$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x = -\cot x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{1+x\sqrt{x^2-1}} = \sec^{-1} x + C$$

$$\int \frac{dx}{x^2+1} = \tan^{-1} x + C$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int k f(x) dx = k \int f(x) dx.$$

$$\int f(x) dx = g(x) + C$$

$$\int f(ax+b) dx = \frac{g(ax+b)}{a} + C$$

$$\int \cos(2x+3) dx = \frac{\sin(2x+3)}{2} + C$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C$$

$$\int \frac{dx}{\sqrt{ax+b}} = (ax+b)^{-1/2} dx = \frac{2(ax+b)^{1/2}}{a}$$

$$\int \frac{dx}{(ax+b)^2} = \int (ax+b)^{-2} dx = \frac{(ax+b)^{-1}}{-a}$$

$$\int e^{\ln \sqrt{x}} dx.$$

$$e^{\ln \sqrt{x}}$$

$$\frac{2x^{3/2}}{3} + C$$

$$\int \frac{\sqrt{x^4 - x^{-4} + 2}}{x^3} dx$$

$$\frac{(x^4 - x^{-4} + 2)^{1/2}}{x^3} \cdot dx$$

$$(x^4 - x^{-4} + 2)^{1/2} x^{-3} dx$$

$$\int \frac{x^2 + x^{-2}}{x^3} dx$$

$$\int \frac{x\sqrt{x+1}}{x+\sqrt{x+1}} dx$$

$$t = \sqrt{x}$$

$$\int \frac{1}{x} + x^{-5} dx$$

$$\ln|x| + \frac{x^{-4}}{-4} + C$$

$$\int \frac{t^3 - 1}{t^2 + t + 1} dt$$

$$\int (\sqrt{x}-1) dx$$

$$\frac{2x^{3/2}}{3} - x + C$$

$$\int \frac{dx}{ax+b} = \frac{\ln|ax+b|}{a} + C$$

$$\int \frac{dx}{a} = \lambda \ln|1| + k$$

$$\int \frac{dx}{x^2 + 3x - 10} = \frac{1}{7} \int \frac{(x+5) - (x-2)}{(x+5)(x-2)}$$

$$= \frac{1}{7} \ln \left| \frac{x-2}{x+5} \right| + C$$

$$\int \frac{dx}{3-2x} = \frac{\ln(3-2x)}{-2} + C$$

$$\int \frac{dx}{2x^2 - x - 1} = \int \frac{du}{2u^2 - 2u + 1 - 1} = \int \frac{du}{2u(u-1) + (u-1)}$$

$$\frac{1}{3} \int \frac{(2u+1) - 2(u-1)}{(u-1)(2u+1)} du = \frac{1}{3} \int \frac{1}{u-1} - 2, \frac{1}{2u+1} du$$

$$= \frac{1}{3} \left[\ln|u-1| - 2 \ln|2u+1| \right]$$

$$\int \frac{5x+9}{x^2 + 3x - 10} dx = \int \frac{5u+9}{(u-2)(u+5)} du = \int \frac{3(x+5) - 2(x-2) - 2}{(x+5)(x-2)} du$$

$$= \int \frac{3}{x-2} + \frac{2}{x+5} - \frac{2}{(x+5)(x-2)} dx = 3 \ln|x-2| + 2 \ln|x+5| - 1$$

$$+ \ln \left| \frac{x-2}{x+5} \right|$$

$$\int \frac{x}{x+1} dx$$

$$\int \frac{2x-3}{3x-2} dx$$

$$= \frac{2}{3} \int \frac{6x-9}{6x-4} dx$$

$$\int \frac{x+1-1}{x+1} dx$$

$$= \frac{2}{3} \int \frac{6x-4-5}{6x-4} dx$$

$$x - \int \frac{1}{x+1} dx$$

$$= \frac{2}{3} \left[x - \frac{5}{6} \ln|6x-4| + C \right]$$

$$x = \ln|x+1|$$

$$\int a^{px+q} dx = \frac{a^{px+q}}{p \ln a} + C$$

$$\int 3^{5x-4} dx = \frac{3^{5x-4}}{(\ln 3)(5)} - C$$

$$\int 3^{\frac{x}{5}-x} dx = \int \left(\frac{3}{5}\right)^u \cdot du = \frac{\left(\frac{3}{5}\right)^u}{\ln\left(\frac{3}{5}\right)}$$

$$\int \frac{a^x + b^x + c^x}{(abc)^x} dx.$$

$$\int \left(\frac{1}{bc}\right)^u + \left(\frac{1}{ac}\right)^u + \left(\frac{1}{ab}\right)^u du$$

~~$$\frac{\left(\frac{1}{bc}\right)^u}{\ln\left(\frac{1}{bc}\right)} + \frac{\left(\frac{1}{ac}\right)^u}{\ln\left(\frac{1}{ac}\right)} + \frac{\left(\frac{1}{ab}\right)^u}{\ln\left(\frac{1}{ab}\right)}$$~~

$$\int a^m x \cdot b^n x \cdot dx$$

$$\int (a^m \cdot b^n)^x dx$$

$$= \frac{a^m \cdot b^n}{\ln(a^m \cdot b^n)} + C$$

$$\int \frac{e^{3x} + e^{5x}}{e^x + e^{-x}} dx$$

$$\int \frac{e^{4x} + e^{6x}}{e^{2x} + 1} dx$$

$$\int e^{4x} dx$$

$$= \frac{e^{4x}}{4} + C$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos^2 \theta = 1 + \cos 2\theta$$

$$2\sin^2 \theta = 1 - \cos 2\theta$$

$$\sin^4 x + \cos^4 x = 1 - \frac{1}{2} \sin^2 2x$$

$$\sin^6 x + \cos^6 x = 1 - \frac{3}{4} \sin^2 2x$$

$$\int \cos(5-4x) dx = \frac{\sin(5-4x)}{-4} + C$$

$$\begin{aligned} \int \cos 2x \cos 3x dx &= \frac{1}{2} \left\{ (\cos(5x) + \cos x) dx \right. \\ &= \frac{1}{2} \left[\frac{\sin 5x}{5} + \sin x \right] + C \end{aligned}$$

$$\int \cos^2 3x dx$$

$$\int (1 + \cos 6x) dx = \int (\cos^2 u)^2 du$$

$$x + \frac{\sin 6x}{6} + C$$

$$\int \cos^4 u du$$

$$= \int (1 + \cos 2u)^2 du$$

$$= \int \frac{(1 + \cos 2x)^2}{4} dx$$

$$= \int (\cos^4 x - \sin^4 x) dx$$

$$= \frac{1}{4} \left[x + \sin 2x + \int \frac{1 + \cos 4x}{2} dx \right]$$

$$= x - \frac{1}{2} \int \sin^2 2x dx$$

$$= \frac{x + \sin 2x}{4} + \frac{1}{8} \left(x + \frac{\sin 4x}{4} \right)$$

$$= x - \frac{1}{2} (1 - \cos 4x) dx$$

$$= x - \frac{1}{2} \left[x - \frac{\sin 4x}{4} \right] + C$$

$$= \int \cos^2 x$$

$$\int \frac{\cos x - \cos 2x}{1 - \cos x} dx$$

$$= \int \frac{2 \cos^2 x - \cos x - 1}{\cos x - 1} dx$$

$$= \int \frac{(\cos x - 1)(2 \cos x + 1)}{(\cos x - 1)} dx$$

$$= 2 \sin x + x + C$$

$$\int (\sin^6 x + \cos^6 x) dx$$

$$\int \sec^2(ax+b) dx = \frac{\tan(ax+b)}{a} + C$$

$$\int \csc^2(ax+b) dx = -\frac{\cot(ax+b)}{a} + C$$

$$\int \sec^2(3x-4x) dx = \frac{\tan(3-4x)}{-4} + C$$

$$\int \frac{dx}{1+\cos x}$$

$$\int \frac{du}{2\cos^2 \frac{u}{2}} = \frac{1}{2} \int \sec^2 \frac{u}{2} du = \frac{2}{2} \tan \frac{u}{2} = \tan \frac{u}{2} + C$$

$$\int \frac{1-\cos 2u}{1+\cos 2u} du$$

$$\int \cot^2 x dx$$

$$\int \frac{2\sin^2 u}{2\cos^2 u} du$$

$$\int (\operatorname{cosec}^2 u - 1) du$$

$$\int (-1 + \sec^2 u) du$$

$$= -\operatorname{Cot} u - u$$

$$\tan u - u + C$$

$$\int \tan^2 x \sin^2 x dx$$

$$= \int (\sec^2 x - 1) \sin^2 x dx$$

$$= \int \tan^2 x - \int \sin^2 x$$

$$= \frac{\tan x - x - 1}{2} \int 1 - \cos 2x dx$$

$$= \tan x - x - \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]$$

$$\int \sec(ax+b) \tan(ax+b) dx = \frac{\sec(ax+b)}{a} + C$$

$$\int \csc(ax+b) \cot(ax+b) dx = -\frac{\csc(ax+b)}{a} + C$$

or

$$\int \frac{dx}{1-\sin^3x} = \int \frac{a \sin^3x + b \cos^3x}{\sin^2x \cos^2x} \cdot dx$$

$$\int \frac{1+\sin^3x}{\cos^2x} dx = a \int \sec x \tan x + b \int \csc x \cot x$$

$$a \sec x + b \csc x + C$$

$$\int \sec^2 3x + \tan 3x \sec 3x$$

$$\frac{\tan 3x}{3} + \frac{\sec 3x}{3} + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \frac{\sin^{-1} x}{a} + C$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + C$$

$$= \frac{1}{a} \cdot \frac{1}{\frac{x}{a} \sqrt{\frac{x^2}{a^2}-1}} \cdot \frac{1}{a}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{25-(2x-3)^2}} = \frac{1}{2} \sin^{-1} \left(\frac{2x-3}{5} \right) + C$$

$$\int \frac{dx}{|2x-1|\sqrt{4x^2-4x}} = \int \frac{du}{|2u-1|\sqrt{(2u-1)^2-1}} = \frac{1}{2} \sec^{-1}(2x-1) + C$$

Differentials

$$y = f(x)$$

$$dy = f'(x) dx$$

$$\tan \theta = \frac{dy}{dx}$$

$$dx \cdot f'(x) = dy$$

$$d(\sin x) = \cos x dx$$

$$d(\ln x) = \frac{1}{x} dx$$

Integration by substitution

$$\int e^x \sin(e^x) dx$$

$$e^x = t$$

$$e^x dx = dt$$

$$\int \sin t dt$$

$$= -\cos t$$

$$= -\cos(e^x)$$

$$\int \cos x^2 x dx$$

$$x^2 = t$$

$$2x \cdot dx = dt$$

$$\int \frac{1}{2} \cos t dt$$

$$\frac{1}{2} \sin t + C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$\sqrt{x} = t$$

$$\frac{1}{\sqrt{x}} dx = dt$$

$$= \int e^t \cdot dt$$

$$= 2e^t$$

$$\int \cos(\sin x) \cos x dx$$

$$\int \frac{(\tan^{-1} x)^2}{1+x^2} dx$$

$$\sin u = t$$

$$\cos u du = dt$$

$$\int \cos t dt$$

$$\sin t + C$$

$$\tan^{-1} u = t$$

$$\frac{1}{1+u^2} du = dt$$

$$\int t^2 dt$$

$$\frac{t^3}{3}$$

$$-\left(\frac{\tan^{-1} u}{3} \right)^3 + C$$

$$\int \frac{\cos(\sin^{-1}x)}{\sqrt{1-x^2}} dx$$

$$\sin^{-1}u = t$$

$$\frac{1}{\sqrt{1-u^2}} \cdot du = dt$$

$$\int \cos(t) dt$$

$$\sin(\sin^{-1}u) + C$$

$$\int \frac{\tan\sqrt{x} \sec^2\sqrt{x}}{\sqrt{x}} dx$$

$$\tan\sqrt{u} = t$$

$$\frac{\sec^2\sqrt{u}}{2\sqrt{u}} \cdot du = dt$$

$$\int \frac{yt}{t^2} dt$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$f(x) = t$$

$$f'(x) dx = dt$$

$$\int \cot x dx = \ln|\sin x| + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\frac{s(t+S)}{(s+t)(S)} = \frac{st+s^2}{s+t} = \ln|\tan(\frac{\pi+x}{2})| + C$$

$$\int \cosec x dx = \ln|\cosec x - \cot x| + C$$

$$= \ln|\frac{\tan x}{2}| + C$$

$$\int \frac{\cos x - \sin x}{\sin x + \cos x} dx$$

$$\int \frac{\cos 2x}{1 + \sin 2x} \cdot dx = \int \frac{1 - \tan u}{1 + \tan u} du$$

$$\ln|\sin x + \cos x| + C$$

$$\int \frac{e^x}{e^x + 1} dx$$

$$\int \frac{e^x + 1 - 1}{e^x + 1} dx$$

$$\int 1 - \frac{1}{e^x + 1} dx$$

$$x - \int \frac{1}{e^x + 1} dx = \ln|e^x + 1| + C$$

$$\int \frac{1}{e^x + 1} dx$$

$$- \int \frac{-e^{-x}}{1 + e^{-x}} dx$$

$$= -\ln(1 + e^{-x}) + C$$

$$= -\ln \left(\frac{e^x + 1}{e^x} \right) + C$$

$$= \{ \ln(e^x + 1) - \ln e^x \} + C$$

$$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$$

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$\ln(e^x + e^{-x}) + C$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} dx$$

$$\int \frac{\sec^4 x}{\sqrt{\tan x}} dx$$

$$= 2\sqrt{f(x)} + C$$

$$\int \frac{1 + \tan^2 x \cdot \sec^2 x}{\sqrt{\tan x}} dx$$

$$\tan x = t$$

$$\sqrt{f(x)} = t$$

$$\int \frac{1 + t^2}{t^2} dt$$

$$2\sqrt{t} + \frac{2}{5} t^{\frac{5}{2}} + C$$

$$\frac{\sin(A - B)}{\cos A \cos B} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\frac{\sin(A - B)}{\sin A \sin B} = \frac{\cot B - \cot A}{1 + \cot A \cot B}$$

$$\int \frac{\sin x}{\sin^2 x \sin^3 x} dx$$

$$\int \frac{\sin(3x-2x)}{\sin^2 x \sin^3 x} dx$$

$$-\int \cot 3x + \int \cot 2x dx$$

$$\frac{\ln(\sin 2x)}{2} - \frac{\ln|\sin 3x|}{3} + C$$

$$\int \frac{dx}{\cos(x-a) \cos(x-b)}$$

$$\int \frac{\sin((x-a)-(x-b))}{\cos(x-a) \cos(x-b)} dx$$

$$\int \sin^2 x \cos^3 x dx$$

$$\int \sin^2 x \cos^2 x \cos x dx$$

b=0

$$\int \frac{\sin 2x}{b \cos^2 x + a \sin^2 x} dx$$

$$b \cos^2 x + a \sin^2 x = 1$$

$$(a-b) \sin 2x dx = dt$$

$$\frac{1}{a-b} \int \frac{dt}{t^{100}}$$

$$\int (\tan^3 x - x \tan^2 x) dx$$

$$\int (\tan x - x) (\tan^2 x) dx$$

$$\tan x - x = t$$

$$\int + dt$$

$$\int \frac{x \cos x dx}{(\alpha \sin x + \cos x)^n}$$

Standard substitution

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin \theta$$

$$\sqrt{x^2 - a^2} \quad x = a \tan \theta$$

$$\sqrt{x^2 - a^2} \quad x = a \sec \theta$$

$$\sqrt{\frac{a-x}{a+x}} \quad x = a \cos \theta$$

$$\int \frac{x^2}{\sqrt{a^6 - x^6}} dx.$$

$$x^3 = a^3 \sin \theta$$

$$3x^2 dx = a^3 \cos \theta d\theta$$

$$\theta = \sin^{-1} \frac{x^3}{a^3}$$

$$\frac{1}{3} \int \frac{a^3 \cos \theta d\theta}{a^3 \cos \theta} = \frac{1}{3} \theta + C$$

$$\frac{1}{3} \sin^{-1} \frac{x^3}{a^3} + C$$

$$\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$$

$$\int \frac{x^{1/2}}{\sqrt{(a^{3/2})^2 - (x^{3/2})^2}} dx$$

$$x^{3/2} = a^{3/2} \sin \theta$$

same as d. above

$$\int \frac{\sqrt{(a-x^2)^3}}{x^6} dx$$

$$x = 3 \sin \theta$$

$$\cos^2 \theta$$

$$dx = 3 \cos \theta d\theta$$

$$\int \frac{3^4}{36} \cdot \frac{\cos^4 \theta}{\sin^6 \theta} d\theta$$