Toppersnotes

Unleash the topper in you

THE IIT - JEE SECRET

JEE MAINS AND JEE ADVANCED

MATHEMATICS - III

Calculus - I

SIERRA INNOVATIONS Private Limited

Contents

7. Application of Derivative 176-235

Tangent and Normal

Monotonicity

Maxima and Minima

FUNCTION

Cartesian product of 2 sets $(A \times B)$

$A \equiv \{a, b, c, d\}$ $\quad$ $A \times B \equiv \{(a,b) \mid a \in A, b \in B\}$

$B \equiv \{p, q\}$

$A \times B = \{(a,p)(a,q) - - - - - (d,p)(d,q)\}$

$n(A) = m \quad n(B) = n$

$n(A \times B) = mn$

If $n(A) = m$, $n(B) = n$

Total no. of subsets of $A \times B = 2^{mn}$

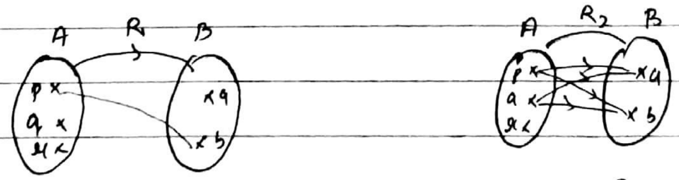
Relation

$A \equiv \{p, q, r\}$

$B \equiv \{a, b\}$

It is defined as from set A to set B is any subset of $A \times B$

$R \subset A \times B \quad ; \quad R \equiv \{(a,b) \mid a \in A, b \in B\}$

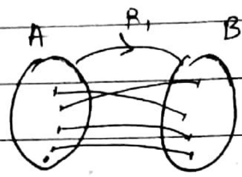


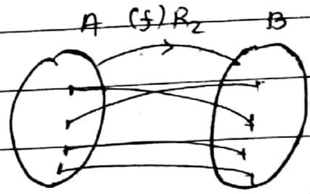
$R_1 \equiv \{(p,b)\}$ $\quad$ $R_2 \equiv \{(p,a), (p,b), (q,a), (q,b)\}$

Function

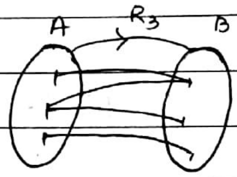
A relation from set A to set B is called if every element of set A is connected to some unique element of set B, it is denoted as

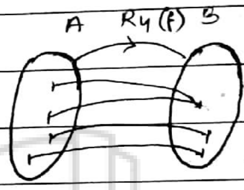
$f : A \to B$





$f : A \to B$

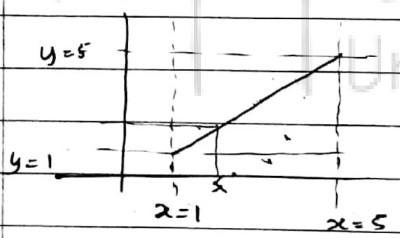




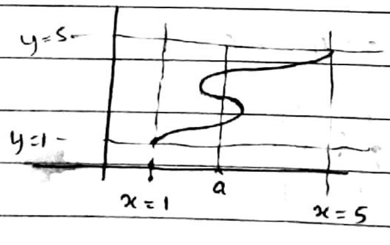
$f : A \to B$

Identify which of the following from A to B represents function.

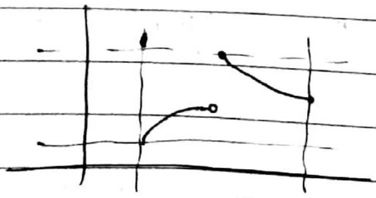
$A = [1, 5]$ $\quad B = [1, 5]$



It is a function



Not a function

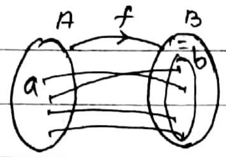


It is a function

Note :-

If a vertical line cuts the given graph in more than 1 point then it can't be graph of function.

$f : A \to B$



$A \to$ domain of the f^n

$B \to$ co-domain of the f^n

$b = f(a)$

b is image of a under f

a is pre-image of b under f

Range of f^n

The set of all f-images is called range of f^n

range $\subseteq$ co-domain

Find the domains of function :-

$$f(x) = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$$

$$\frac{(x+1)(x-3)}{(x-2)} \geq 0$$

$-$ $+$ $-$ $+$

-1 2 3

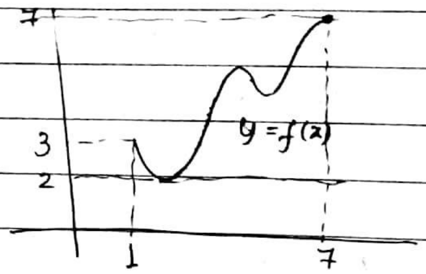
$D_f \; [-1, 2) \cup [3, \infty)$

$$f(x) = \log_{(7-x)}(x-5)$$

$7 - x > 0$ and $7 - x \neq 1$ and $x - 5 > 0$

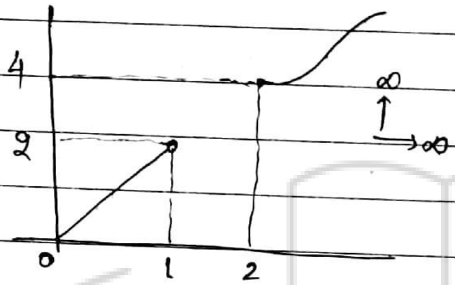
$x < 7$ $\qquad$ $x \neq 6$ $\qquad$ $x > 5$

$D_f \; (5, 6) \cup (6, 7)$



$D_f \in [1, 7]$

$R_f \in [2, 7]$



$f : [0,1) \cup [2,\infty) \rightarrow R$

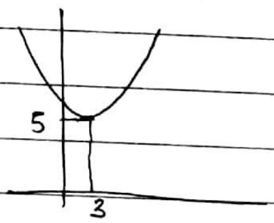
$R_f \quad [0,2) \cup [4,\infty)$

Find the range of following functions:-

$f : R \rightarrow R$

$f(x) = x^2 - 6x + 14$

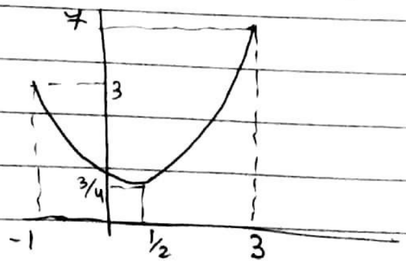
$f(x) = [5, \infty) \quad \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$



$f : [-1, 3] \rightarrow R$

$f(x) = x^2 - x + 1$

vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$



$R_f \quad \left[\frac{3}{4}, 7\right]$

$f: R \to R$

$f(x) = 8 - 3\sin x$

$-1 \leq \sin x \leq 1$

$-3 \leq 3\sin x \leq 3$

$5 \leq 8 - 3\sin x \leq 11$

$f(x) = \frac{1}{\sqrt{9 + 5\sin x}}$

$-5 \leq \sin x \leq 5$

$4 \leq 9 + 5\sin x \leq 14$

$2 \leq \sqrt{9 + 5\sin x} \leq \sqrt{14}$

$\frac{1}{\sqrt{14}} \leq \frac{1}{\sqrt{9+5\sin x}} \leq \frac{1}{2}$

$\left[\frac{1}{\sqrt{14}}, \frac{1}{2}\right]$

Note:-

$-2 < x < 5$

$-2 < x < 0$ or $0 < x < 5$

$-\infty < \frac{1}{x} < -\frac{1}{2}$ or $\frac{1}{5} < \frac{1}{x} < \infty$

Some Important Functions

→ Polynomial Functions

$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} \text{ ------ } + a_n$

$a_0 \neq 0$, $n \in N$

'f' is a polynomial of degree 'n'

Some notes

The graph of poly. f^n is always continous.

The range of odd degree polynomial is R $((-\infty, \infty))$

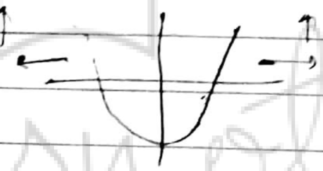
$$f(x) = x^7 - 5x^4 + x^2 + x + 1$$
$$= x^7\left(1 - \frac{5}{x^3} + \frac{1}{x^5} + \frac{1}{x^6} + \frac{1}{x^7}\right)$$

$x \to \infty \quad f \to \infty$

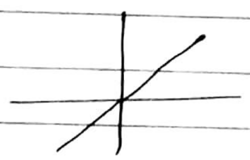
$x \to -\infty \quad f \to -\infty$

The range of even degree polynomial can't be R

$$f(x) = 2x^6 - x^5 + 3x^2 + x + 1$$



The function of form $f(x) = ax$, $a \neq 0$, ~~can not be 0~~ is called add lined polynomial.



The function of form $f(x) = ax+b$, $a \neq 0$ is called linear poynomial.

Algebraic functions

A function f is called algebraic function if is can be constructed using algebraic operations such as $+, -, \times, \div$ and using radical signs.

Examples:- $f = x^2 - 2x + 3$

$f = \sqrt{x}$

$f(x) = |x|$

$f(x) = \sqrt{x^2}$

$$f(x) = \frac{x^2 + x^3 + x - 3x^{3/2}}{\left(\sqrt{x^2+x+1}\right) \cdot 2x^{-1}}$$

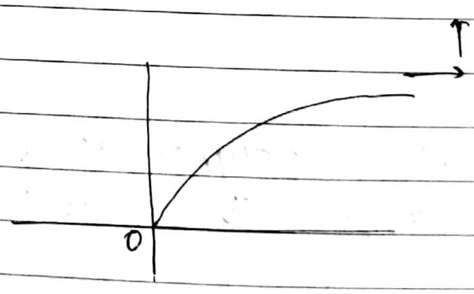
All polynomial functions are algebraic but not ~~conserv~~ converse.

Functions which are not algebraic are called transendental functions.

$f(x) = \sqrt{x}$

$D_f \rightarrow R^+ \cup \{0\}$

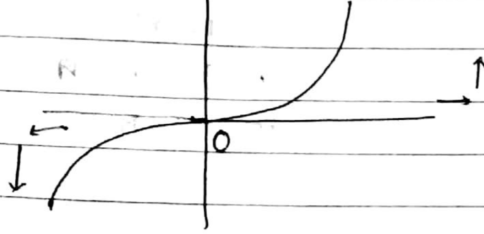
$R_f \rightarrow [0, \infty)$



$f(x) = x^3$

$R_f \rightarrow (-\infty, \infty)$

$D_f \rightarrow R$



Rational Functions

The functions of the form $f(x) = \dfrac{g(x)}{h(x)}$ where $g(x)$ & $h(x)$ are polynomial functions are called rational functions.

Examples -

$$f(x) = \frac{x-1}{x-2}$$

$$f(x) = \frac{x^2-x+1}{x^2+x+1}$$

$$f(x) = \frac{1}{x} \qquad D_f \rightarrow R-\{0\}$$

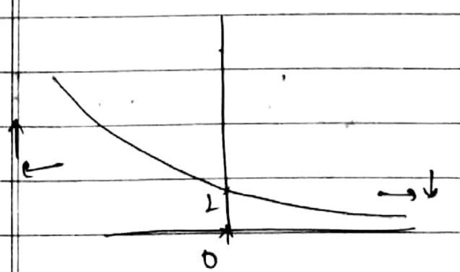
$$y = \frac{1}{x} \qquad x = \frac{1}{y} \qquad R-\{0\} \rightarrow \text{range}$$

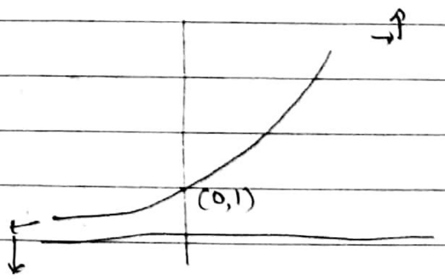
Exponential functions

$$f(x) = a^x \qquad a \neq 1 \text{ and } a > 0$$

$0 < a < 1$ — $f(x) = a^x$

$a > 1$ — $f(x) = a^x$



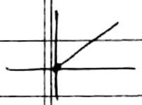


$$D_f \rightarrow R$$

$$R_f \rightarrow R^+$$

	R_f	D_f
$y = 2^{x^3}+1$	$(1,\infty)$	R
$y = 2^{\tan x}$	R^+	$R - (2n-1)\pi/2$
$y = 2^{\sin x}$	$[\frac{1}{2}, 2]$	R
$f(x) = e^{\ln x}$	R^+	R^+

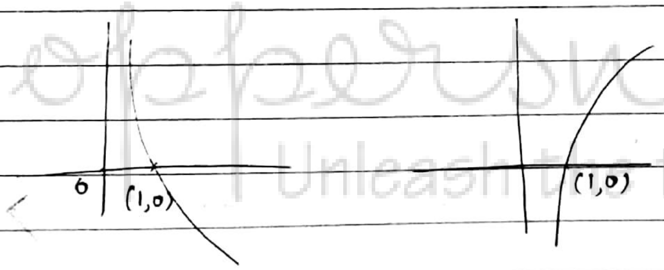
$= x$



Logarithm functions

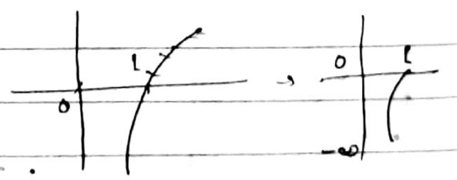
$f(x) = \log_a x$ $\qquad D_f \to R^+$

$0 < a < 1$ $\qquad\qquad a > 1$



fn	D_f	R_f
$f(x) = \log x^2$	$R - \{0\}$	R
$f(x) = \log\lvert\sin x\rvert$	$R - n\pi$	$(-\infty, 0]$

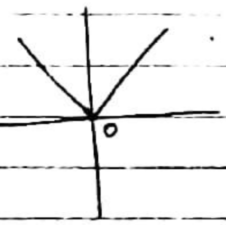
$\log(\quad)$
$\uparrow$ $(0,1]$
$\lvert\sin x\rvert$
$\uparrow$
x



Absolute Value function [Modulus functions]

$$f(x) = |x| = \begin{cases} \to -x, & x<0 \\ \to x, & x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} -x, & x<0 \\ x, & x \geq 0 \end{cases}$$ non uniform fn



$D_f \to R$

$R_f \to [0, \infty)$

$f(x) = \frac{1}{|x|}$ $\quad D_f \to R - \{0\}$

$y = \frac{1}{|x|}$

$|x| = \frac{1}{y}$

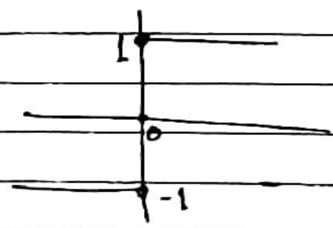
$\frac{1}{y} \geq 0$ $\quad R_f \to R^+$

Signum functions

$$f(x) = Sgn(x) = \begin{cases} -1 & x<0 \\ 0 & x=0 \\ 1 & x>0 \end{cases}$$

$D_f \to R$

$R_f = \{-1, 0, 1\}$



$f(x) = Sgn(x^2+x+1)$

$D_f \to R$, $R_f \to 1$

Greatest Integer functions

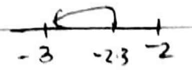
$f(x) = [x]$

$x = I + f \qquad I \to \text{int}$

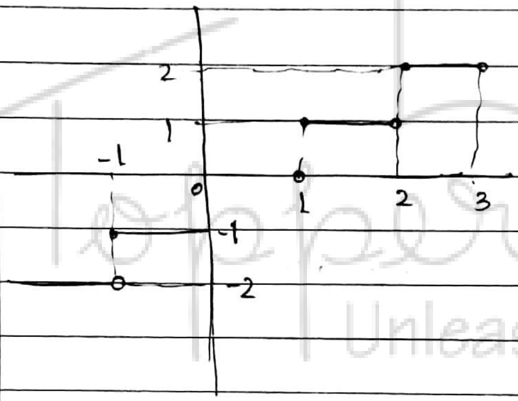
$[x] = I \qquad 0 \le f < 1$

$[1.9999] = 1$

$[-2.3] = -3$



$$f(x) = [x] = \begin{cases} -1 & -1 \le x < 0 \\ 0 & 0 \le x < 1 \\ 1 & 1 \le x < 2 \end{cases}$$



Step-up function

$D_f \to I$

$[x] = 2$

$2 \le x < 3$

$[2, 3)$

$2[x] = 3$

N.S.

$[x] = 0$

$0 \le x < 1$

$[x] = -2$

$-2 \le x < -1$

$[x] < 4$

$x < 4$

$(-\infty, 4)$

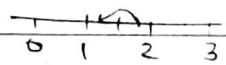
$[x] \le 4$

$-\infty < x < 5$

$(-\infty, 5)$

$1 < [x] < 3$

$2 \le x < 3$



Properties of GIF

* (a) $[x+m] = [x] + m \quad ; \quad m \in I$

$\underset{I+f}{\downarrow}$

* (b) $[x] + [-x] = \begin{cases} 0 & ; \; x \in I \\ -1 & ; \; x \notin I \end{cases}$

$\underset{\underset{\cancel{f}}{I+f}}{\downarrow} \qquad \underset{\underset{\underset{-1-\cancel{I}}{-1-I+1-f}}{-I-f}}{|}$

(c) $[x] \leq x < [x] + 1$

$I \leq I + f \leq I + 1$

(d) $[x] + [y] \leq [x+y] \leq [x] + [y] + 1$

$I_1 + I_2 + f_1 + f_2$

$0 \leq f_1 < 1$

$0 \leq f_2 < 1$

$0 \leq f_1 + f_2 < 2$

Find the domain and range of f^n $f(x) = \dfrac{1}{[x]}$

$f(x) = \dfrac{1}{[x]}$

$[x] = 0$

$0 \leq x < 1$

$[0, 1)$

$D_f \rightarrow R - [0, 1)$

$(-\infty, 0) \cup [1, \infty)$

$R_f = \left\{ \dfrac{1}{N} \,\middle|\, N \in I, N \neq 0 \right\}$