Toppersnotes
Unleash the topper in you

THE IIT - JEE SECRET

JEE MAINS AND JEE ADVANCED

MATHEMATICS - I

Algebra & Trigonometry

SIERRA INNOVATIONS
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Contents

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LOGARITHM

Defination

$a^x = N \quad (a>0, a\neq 0, N>0)$

'x' is logarithm of N to base 'a'

$x = \log_a N \quad (a>0, a\neq 0, N>0)$

Asking

$\log_2 8 = 3$

$\log_2 x = 4$

$x = 16$

$\log_5 125 = 3$

$\log_x 1 = 0$

Identities

$\log_a 1 = 0$

$\log_{1/x} x = -1$

$\log_x x = 1$

Questions

a) $\log_{\frac{4}{3}}(1.\overline{3})$

$\Rightarrow 1$

b) $\log_{10}(\cos 0^\circ)$

$\log_{10} 1 = 0$

c) $\log_{0.125} 8$

$\Rightarrow \log_{\left(\frac{1}{8}\right)} 8$

$\Rightarrow -1$

d) $\log_{3\sqrt{7}}(2401)$

$\Rightarrow \log_{3\sqrt{7}}(7)^4$

$\Rightarrow 12$

d) $\log_{5\sqrt{5}} 125$

$= 2$

e) $(\log(\tan 1^\circ))(\log(\tan 2^\circ)) \ldots (\log(\tan 89^\circ))$

$(\log(\tan 1^\circ))(\log(\cot 1^\circ)) \ldots$

$\Rightarrow 0$

$(\log(\sin 1^\circ)) \ldots (\log(\sin 90^\circ)) = 0$

$\Rightarrow 0$

NOTE :-

$\log x = \log_e x = \ln x$ (Natural log.)

PROPERTIES

(1) $\log_a(MN) = \log_a M + \log_a N$

Let $\log_a M = x$ $\quad a^x = M$

$\log_a N = y$ $\quad a^y = N$

$mn = a^{x+y}$

$\log_a mn = x + y = \log_a m + \log_a N$

(2) $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

Let $\log_a M = x$ $\quad a^x = M$

$\log_a N = y$ $\quad a^y = N$

$\frac{M}{N} = \frac{a^x}{a^y}$

$\frac{M}{N} = a^{x-y}$

$\log_a\left(\frac{M}{N}\right) = x - y$

$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

(3) $\log_a (M)^p = p \log_a (m)$

$\log_a m = x$

$\log_a m = x$

$m = a^x$

$m^p = a^{xp}$

$\log_a (m)^p = xp = p \log_a m$

Simplify the following :-

(a) $\log_2 (2)^3 \Rightarrow 3 \log_2 (2)$

$\Rightarrow 3$

(b) $\log_2 10 + \log_2 5 - \log_2 25 + 1$

$\Rightarrow \log_2 50 - \log_2 25 + 1$

$\Rightarrow \log_2 \left(\frac{50}{25}\right) + 1$

$\Rightarrow \log_2 (2) + 1$

$\Rightarrow +1 + 1$

$\Rightarrow 2$

(c) $\log_5 \left(\frac{15}{4}\right) + \log_5 \left(\frac{16}{25}\right) + \log_5 \left(\frac{1}{12}\right)$

$\Rightarrow \log_5 \left(\frac{\overset{3}{15}}{4} \times \frac{\overset{4}{16}}{\underset{5}{25}}\right) + \log_5 \left(\frac{1}{12}\right)$

$\Rightarrow \log_5 \left(\frac{12}{5}\right) + \log_5 \left(\frac{1}{12}\right)$

$\Rightarrow \log_5 5 = 1$

$\log_a\left(\frac{a^{5/7}\sqrt{b}}{c^2\sqrt[3]{d^5}}\right)$ $\log_a b = 2$, $\log_a c = 1$, $\log_a d = 3$

$\Rightarrow \log_a\left(\frac{a^{5/7}\sqrt{b}}{c^2\sqrt[3]{d^5}}\right)$

$\Rightarrow \log_a\left(\frac{a^{5/7}\sqrt{b}}{c^2 d^{5/3}}\right)$

$\Rightarrow \log_a\left(\frac{a^{5/7} b^{1/2}}{c^2 d^{5/3}}\right)$

$\Rightarrow \log_a\left(a^{5/7} b^{1/2}\right) - \log_a\left(c^2 d^{5/3}\right)$

$\Rightarrow \log_a\left(\log_a a^{5/7} + \log_a b^{(1/2)}\right) - \log_a\left(\log_a c^2 + \log_a d^{5/3}\right)$

$\Rightarrow \log_a\left(\frac{5}{7}\log_a a + \frac{1}{2}\log_a b\right) - \log_a\left(2\log_a c + \frac{5}{3}\log_a d\right)$

$\Rightarrow \log_a\left(\frac{5}{7}+1\right) - \log_a(2+5)$

$\Rightarrow \log_a\left(\frac{12}{7}\right) - \log_a(7)$

$\Rightarrow \log_a\left(\frac{\frac{12}{7}}{7}\right)$

$\Rightarrow \log_a \frac{12}{49}$

$\Rightarrow \frac{37}{7}$

(a) $2\log\left(\frac{8}{45}\right) + 3\log\left(\frac{25}{8}\right) - 4\log\left(\frac{5}{6}\right) = K\log 2$

Find K.

$2\log\left(\frac{8}{25}\right) + 3\log\left(\frac{25}{8}\right) - 4\log\left(\frac{5}{6}\right) = K\log 2$

$\Rightarrow$ ~~2 log~~ $(\log_e 8 - \log_e 25) + 3\log(\log_e e^{25} - \log_e 8)$
$- 4\log(\log_e 5 - \log_e 6)$

$\Rightarrow$ ~~2 log~~ $(\log_e (2)^3 - \log_e (5)^2) + 3\log(\log_e (5)^2 - \log_e (2)^3$
$- 4\log(\log_e 5 - \log_e 6)$

$\Rightarrow$ ~~2 log 10~~ $(\log_{10}(2)^3 - \log_{10}(5)^2) + 3\log_{10}(\log_{10}(5)^2 - \log_{10}(2)^3$
$- 4\log_{10}(\log_{10} 5 - \log_{10} 6)$

$\Rightarrow$ $(3\log_{10} 2 - 2\log_{10} 5) + (2\log_{10} 2 - 3\log_{10} 2)$
$-(\log_{10} 5 - \log_{10} 6)$

$2\log\left(\frac{8}{45}\right) + 3\log\left(\frac{25}{8}\right) - 4\log\left(\frac{5}{6}\right)$

$= 2\log\left(\frac{2^3}{3^2 \cdot 5}\right) + 3\log\left(\frac{5^2}{3^3}\right) - 4\log\left(\frac{5}{2 \cdot 3}\right)$

$= 2(3\log 2 - 2\log 3 - \log 5) + 3(2\log 5 - 3\log 2)$
$- 4(\log 5 - \log 2 - \log 3)$

$\log 2 = K\log 2$

$K = 1$

(b) $\log_a \sqrt[4]{a^{815}} = ?$

(c) $\log_{10}\left(\sqrt{a^{-2}.b}\right)\left(\sqrt[3]{ab^{-3}}\right)$

$\Rightarrow \log_{10}(a^{-2}.b)^{1/2}\ (ab^{-3})^{1/3}$

$\Rightarrow \left(\log_{10}a^{-2} + \log_{10}b\right)^{1/2}\left(\log_{10}a + \log_{10}b^{-3}\right)^{3/2}$

$\Rightarrow \left(-2\log_{10}a + \log_{10}b\right)^{1/2}\left(\log_{10}a + (-3)\log_{10}b\right)^{3/2}$

$\Rightarrow \log_{10}(a^{-1}.b^{1/2}\ a^{1/3}.b^{-1})$

$\Rightarrow \log_{10}(a^{-2/3}.b^{-1/2})$

$\Rightarrow \log_{10}a^{-2/3} + \log_{10}b^{-1/2}$

Property - 4

$a^{\log_a b} = b$

Proof $\rightarrow \log_a b = x$

$a^x = b$

$a^{\log_a b} = b$

Base changing Theorem

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$\log_c a = x$

$\log_c b = y$

$a = c^x \quad -①$

$b = c^y \quad -②$

$c = b^{1/y}$

$a = (b)^{x/y}$

$\log_b a = \frac{x}{y}$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Examples- $\log_{125} 625 = \frac{\log_5 625}{\log_5 125} = \frac{4}{3}$

$$\frac{\log_3 16}{\log_3 4} = \log_4 16 = 2$$

$$\log_{(b^k)} a = \frac{1}{k} \log_b a$$

$$= \frac{\log_b a}{\log_b (b^k)}$$

$$= \frac{\log_b a}{k \log_b b} = \frac{1}{k} \log_b a$$

$$a^{\log_c b} = b^{\log_c a}$$

Proof → $a^{\log_c b} = \left(a^{\log_a b}\right)^{\frac{\log_c b}{\log_a b}}$

$$= (b)^{\frac{\log_b a}{\log_b c}}$$

$$= b^{\log_c a}$$

$$\log_b a = \frac{1}{\log_a b}$$

Proof → $\log_b a = \frac{\log_a a}{\log_a b}$

$$= \frac{1}{\log_a b}$$

$a^x = b$

$\log_a b = x$, $a > 0$, $a \neq 1$ and $b > 0$

$\log_a M + \log_a N = \log_a MN$

$\log_a M - \log_a N = \log_a \left(\frac{M}{N}\right)$

$\log_a (m)^p = p \log_a m$

$a^{\log_a b} = b$

$\log_b a = \frac{\log_c a}{\log_c b}$

$$a^{\log_c b} = b^{\log_c a}$$

$$\log_b a = \frac{1}{\log_a b}$$

The bases of log should be same.

Find the value :-

(a) $\log_3 (8\sqrt{2})\ \log_{81}(216)\ \log_4 (27\sqrt{3})\ \log_{6\sqrt{6}}(243)$

$\Rightarrow \log_3 (8\times 2^{1/2})\ \log_{81}(6)^3\ \log_4 (27\times 3^{1/2})\ \log_{6\sqrt{6}}(3)^5$

$\Rightarrow \frac{\log(2)^{7/2}}{\log 3} \times \frac{\log 6^3}{\log 3^4} \times \frac{\log (3)^{7/2}}{\log (2)^2} \times \frac{\log (3)^5}{\log (6)^{3/2}}$

$\Rightarrow \frac{7}{2} \times \frac{\log 2}{\log 3} \times \frac{3}{4} \times \frac{\log 6}{\log 3} \times \frac{7}{2} \frac{\log 3}{2\log 2} \times \frac{5\log 3}{\frac{3}{2}\log 6}$

$= \frac{7}{2} \times \frac{1}{4} \times 7 \times 5$

$27^{\log_3 5}$

$\Rightarrow (3)^{3\log_3 5}$

$\Rightarrow 5^3 = 125$

$2^{\log_3 5} - 5^{\log_3 2}$

$\Rightarrow 5^{\log_3 2} - 5^{\log_3 2}$

$\Rightarrow 0$

Prove the following :-

$$\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8} = 0$$

$$\log_2 3 + 2 \times \frac{1}{\log_9 4} - 3 \times \frac{1}{\log_{27} 8} = 0$$

$$\log_2 3 + 2\log_4 9 + 3\log_8 27 = 0$$

$$\frac{\log_2 3}{\log_2 2} + \frac{\cancel{2}\log_2 9}{\cancel{2}} - \frac{\cancel{3}\log_2 27}{\cancel{3}} = 0$$

$$\log_2 27 - \log_2 27 = 0$$

$$\frac{\log_3 12}{\log_{36} 3} - \frac{\log_3 4}{\log_{108} 3} = 2$$

$$\Rightarrow \frac{\log_3 12}{\frac{1}{\log_3 36}} - \frac{\log_3 4}{\frac{1}{\log_3 108}}$$

$$\Rightarrow (\log_3 12)(\log_3 36) - (\log_3 4)(\log_3 108)$$

$$\Rightarrow (1 + 2\log_3 2)(2 + 2\log_3 2) - (2\log_3 2)(3 + 2\log_3 2)$$

$$\Rightarrow 2 + 6\log_3 2 + 4(\log_3 2)^2 - 6\log_3 2 - 4(\log_3 2)^2$$

$$\Rightarrow 2$$

If $a = \log 2$, $b = \log 3$ and $c = \log 5$ then express the following in terms of a, b, c.

(a) $\log 12$

$\log 48$

$\log\left(\frac{25}{25}\right)$

$\log 12$

$\log(2^2 \times 3)$

$\log 2^2 + \log 3$

$2\log 2 + \log 3$

$2a + b$

$\log 48$

$\log(2^2 \times 3 \times 3)$

$2\log 2 + 2\log 3$

$2a + 2b$

$4a + b$