



# THE IIT - JEE SECRET

JEE MAINS AND JEE ADVANCED

MATHEMATICS - I  
Algebra & Trigonometry



# Contents

<b>1. Logarithm</b>	<b>01-28</b>
a. Introduction	1
b. Properties of logarithm	3
c. Base changing theorem	8
d. Intervals	14
e. Modulus	15
f. Characteristic and mantissa	18
g. Antilog	28
<b>2. Trigonometric ratio and Identities</b>	<b>29-98</b>
a. Basic formulas	29
b. Angles (degree and radian)	38
c. Trigonometric values (0-90)	39
d. Reduction Formulas	42
e. Trigonometric Identities	44
f. Continued Product of cos	56
g. Values of special angle (22.5, 18, 36)	62
h. Conditional trigonometric identities	67
i. Graph of trigonometric function	72
j. Range of trigonometric function	78
k. Summation of trigonometric series	88
l. Trigonometric inequalities	95
<b>3. Trigonometric Equation</b>	<b>99-116</b>
a. Principal & general solution	99
b. Method of solving trigonometric equation	104
c. Solving system of trigonometric equation	105
d. Trigonometric inequalities	113

## **4. Quadratic Equation** **117-196**

a. Definition & roots of quadratic equation	117
b. Sum & product of roots	118
c. Nature of roots	121
d. Graph & range of Quadratic equation	124
e. Quadratic identity	132
f. Algebraic inequalities	137
g. Condition of common roots	144
h. Maximum and minimum value of quadratic & rational function	151
i. Theory of equation	160
j. Location of roots	167
k. Miscellaneous Problems	179
l. Modulus function & inequalities	183
m. Logarithmic inequalities	190

## **5. Sequence and Series** **197-250**

a. Introduction & definition	197
b. Arithmetic Progression	198
c. Sum of n terms of an AP	199
d. Arithmetic mean	208
e. Geometric Progression	214
f. Sum of n terms of an GP	215
g. Geometric mean	223
h. Arithmetic geometric progression	229
i. Summation of miscellaneous series	233
j. Harmonic Progression	241
k. Harmonic mean	244

## **6. Binomial Theorem** **251-279**

a. General term in the expansion of $(x + y)^n$	251
b. Middle term in the expansion of $(x + y)^n$	257
c. Numerically greatest term	258
d. Solution of series	262
e. Approximation	270
f. Exponential series	271
g. Logarithmic series	274
h. Multinomial theorem	277

## **7. Permutation and Combination** **280-317**

a. General introduction	280
b. Fundamental principal of counting	282
c. Factorial	287
d. Permutation	289
e. Combination	289
f. Formation of groups	296
g. Permutation of alike objects	298
h. Circular Permutation	303
i. Distribution of alike objects to different receiver	312
j. Derangements	315
k. Multinomial theorem	317

# LOGARITHM

Definition

$$a^x = N \quad (a > 0, a \neq 1, N > 0)$$

'x' is logarithm of N to base 'a'

$$x = \log_a N \quad (a > 0, a \neq 1, N > 0)$$

Asking

$$\log_2 8 = 3$$

$$\log_2 x = 4$$

$$x = 16$$

$$\log_5 125 = 3$$

$$\log_x 1 = 0$$

Identities

$$\log_a 1 = 0$$

$$\log_{\frac{1}{a}} x = -1$$

$$\log_a a^x = x$$

### Questions

a)  $\log_{\frac{1}{3}}(1.3)$   
 $\Rightarrow f$

b)  $\log_{10}(\cos 0^\circ)$   
 $\log_{10} 1 = 0$

c)  $\log_{0.125} 8$   
 $\Rightarrow \log_{\left(\frac{1}{8}\right)^3} 8$   
 $\Rightarrow -1$

d)  $\log_{3\sqrt{7}}(240)$   
 $\Rightarrow \log_{3\sqrt{7}}(3\sqrt{7})^4$   
 $\Rightarrow 12$

~~d)~~  $\log_{5\sqrt{5}} 125$   
 $= 2$

e)  $(\log(\tan 1^\circ)) (\log(\tan 2^\circ)) \dots (\log(\tan 89^\circ))$   
 $(\log(\tan 1^\circ)) (\log(\cot 1^\circ)) \dots$   
 $\Rightarrow 0$

$(\log(\sin 1^\circ)) \dots (\log(\sin 90^\circ)) = 0$   
 $\Rightarrow 0$

Note :-

$$\log x = \log_a x = \ln x \quad (\text{Natural log.})$$

### PROPERTIES

$$(1) \log_a(MN) = \log_a M + \log_a N$$

Let  $\log_a M = x \quad \log_a N = y$   
 $a^x = M \quad a^y = N$

$$MN = a^{x+y}$$

$$\log_a MN = x+y = \log_a M + \log_a N$$

$$(2) \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

Let  $\log_a M = x \quad \log_a N = y$   
 $a^x = M \quad a^y = N$

$$\frac{M}{N} = \frac{a^x}{a^y}$$

$$\frac{M}{N} = a^{x-y}$$

$$\log_a\left(\frac{M}{N}\right) = x-y$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$(3) \quad (\log_a(m))^p = p \log_a(m)$$

$\log_a m = x$

$$\log_a m = x$$

$$m = a^x$$

$$m^p = a^{xp}$$

$$\log_a(m)^p = xp = p \log_a m$$

Simplify the following :-

$$(a) \quad \log_2(2)^3 \Rightarrow 3 \log_2(2)$$

$\Rightarrow 3$

$$(b) \quad \log_2 10 + \log_2 5 - \log_2 25 + 1$$

$$\Rightarrow \log_2 50 - \log_2 25 + 1$$

$$\Rightarrow \log_2 \left(\frac{50}{25}\right) + 1$$

$$\Rightarrow \log_2 (2) + 1$$

$$\Rightarrow +1 + 1$$

$$\Rightarrow 2$$

$$(c) \quad \log_5 \left(\frac{15}{4}\right) + \log_5 \left(\frac{16}{25}\right) + \log_5 \left(\frac{1}{12}\right)$$

$$\Rightarrow \log_5 \left(\frac{15}{4} \times \frac{16}{25} \times \frac{1}{12}\right)$$

$$\Rightarrow \log_5 \left(\frac{12}{5}\right) + \log_5 \left(\frac{1}{12}\right)$$

$$\Rightarrow \log_5 5 = 1$$

$$\log_a \left( \frac{a^{5/2} \sqrt{b}}{c^2 \sqrt[3]{d^5}} \right)$$

$$\log_a b = 2$$

$$\log_a c = 1$$

$$\log_a d = 3$$

$$\Rightarrow \log_a \left( \frac{a^{5/2} \sqrt{b}}{c^2 \sqrt[3]{d^5}} \right)$$

$$\Rightarrow \log_a \left( \frac{a^{5/2} \sqrt{b}}{c^2 d^{5/3}} \right)$$

$$\Rightarrow \log_a \left( \frac{a^{5/2} b^{1/2}}{c^2 d^{5/3}} \right)$$

$$\Rightarrow \log_a (a^{5/2} b^{1/2}) - \log_a (c^2 d^{5/3})$$

$$\Rightarrow \log_a (\log_a^{5/2} + \log_b^{1/2}) -$$

$$\log_a (\log_a c + \log_d^{5/3})$$

$$\Rightarrow \log_a \left( \frac{5}{7} \log_a a + \frac{1}{2} \log_a b \right)$$

$$- \log_a (2 \log_a c + \frac{5}{3} \log_a d)$$

$$\Rightarrow \log_a \left( \frac{5}{7} + 1 \right) - \log_a (2 + 5)$$

$$\Rightarrow \log_a \left( \frac{12}{7} \right) - \log_a (7)$$

$$\Rightarrow \log_a \left( \frac{12}{49} \right)$$

$$\Rightarrow \log_a \frac{12}{49}$$

$$\Rightarrow \frac{37}{7}$$

$$(a) \quad 2 \log\left(\frac{8}{25}\right) + 3 \log\left(\frac{25}{8}\right) - 4 \log\left(\frac{5}{6}\right) = k \log 2$$

Find  $k$ .

$$2 \log\left(\frac{8}{25}\right) + 3 \log\left(\frac{25}{8}\right) - 4 \log\left(\frac{5}{6}\right) = k \log 2$$

$$\Rightarrow \cancel{2} (\log 8 - \log 25) + 3 \log (\log e^{25} - \log e 8) \\ - 4 \log (\log e 5 - \log e 6)$$

$$\Rightarrow \cancel{2} \log (\log_e(2)^3 - \log_e(5)^2) + 3 \log (\log_e(5)^2 - \log_e(2)^3) \\ - 4 \log (\log_e 5 - \log_e 6)$$

$$\Rightarrow \cancel{2} \log_{10} (\log_{10}(2)^3 - \log_{10}(5)^2) + 3 \log_{10} (\log_{10}(5)^2 - \log_{10}(2)^3) \\ - 4 \log_{10} (\log_{10} 5 - \log_{10} 6)$$

$$\Rightarrow (3 \log_{10} 2 - 2 \log_{10} 5) + (2 \log_{10} 2 - 3 \log_{10} 2) \\ - (\log_{10} 5 - \log_{10} 6)$$

$$2 \log\left(\frac{8}{25}\right) + 3 \log\left(\frac{25}{8}\right) - 4 \log\left(\frac{5}{6}\right)$$

$$= 2 \log\left(\frac{2^3}{3^2 \cdot 5}\right) + 3 \log\left(\frac{5^2}{3^3}\right) - 4 \log\left(\frac{5}{2 \cdot 3}\right)$$

$$= 2(3 \log 2 - 2 \log 3 - \log 5) + 3(2 \log 5 - 3 \log 2) \\ - 4(\log 5 - \log 2 - \log 3)$$

$$\log 2 = k \log 2$$

$$k = 1$$

$$(b) \log_a^4 \sqrt{a^{8/5}} = ?$$

$$(c) \log_{10} (\sqrt{a^{-2} \cdot b}) (3\sqrt{ab^{-3}})$$

$$\Rightarrow \log_{10} (a^{-2} \cdot b)^{1/2} (3(ab^{-3})^{1/3})$$

$$\Rightarrow (\log_{10} a^{-2} + \log_{10} b)^{1/2} (\log_{10} a + \log_{10} b^{-3})^{3/2}$$

$$\Rightarrow (-2 \log_{10} a + \log_{10} b)^{1/2} (\log_{10} a + (-3) \log_{10} b)$$

$$\Rightarrow \log_{10} (a^{-1} \cdot b^{1/2} a^{1/3} \cdot b^{-1})$$

$$\Rightarrow \log_{10} (a^{-2/3} \cdot b^{-1/2})$$

$$\Rightarrow \log_{10} a^{-2/3} + \log_{10} b^{-1/2}$$

Property - 4

$$a^{\log ab} = b$$

$$\text{Proof} \rightarrow \log ab = x$$

$$a^x = b$$

$$a^{\log ab} = b$$

## Base changing theorem

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log_c a = x$$

$$\log_c b = y$$

$$a = c^x \quad \text{--- (1)}$$

$$b = c^y \quad \text{--- (2)}$$

$$c = b^{1/y}$$

$$a = (b)^{x/y}$$

$$\log_b a = \frac{x}{y}$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Example -  $\log_{125} 625 = \frac{\log_5 625}{\log_5 125} = \frac{4}{3}$

$$\frac{\log_3 16}{\log_3 4} = \log_4 16 = 2$$

$$\log_{(b^k)} a = \frac{1}{k} \log_b a$$

$$= \frac{\log_b a}{\log_b (b^k)}$$

$$= \frac{\log_b a}{k \log_b b} = \frac{1}{k} \log_b a$$

$$a^{\log_c b} = b^{\log_c a}$$

Proof  $\rightarrow a^{\log_c b} = (a^{\log_a b})^{\frac{\log_c b}{\log_a b}}$

$$= (b)^{\frac{\log_b a}{\log_b c}}$$

$$= b^{\log_c a}$$

$$\log_b a = \frac{1}{\log_a b}$$

Proof  $\rightarrow \log_b a = \frac{\log_a a}{\log_a b}$

$$= \frac{1}{\log_a b}$$

$$a^x = b$$

$$\log_a b = x, \quad a > 0, \quad a \neq 1 \quad \text{and} \quad b > 0$$

$$\log_a M + \log_a N = \log_a MN$$

$$\log_a M - \log_a N \leftarrow \log_a \left(\frac{M}{N}\right)$$

$$\log_a (m)^p = p \log_a m$$

$$a^{\log_a b} = b$$

$$\log_b a = \frac{\log_a a}{\log_a b}$$

$$a^{\log b} = b^{\log a}$$

$$\log_b a = \frac{1}{\log_a b}$$

The bases of log should be same.

Find the value :-

$$(a) \log_3(8\sqrt{2}) \log_8(216) \log_4(27\sqrt{3}) \log_{6\sqrt{6}}(2^{43})$$

$$\Rightarrow \log_3(8 \times 2^{1/2}) \log_8(6)^3 \log_4(27 \times 3^{1/2}) \log_{6\sqrt{6}}(3)^5$$

$$\Rightarrow \frac{\log(2)^{7/2}}{\log 3} \times \frac{\log 6^3}{\log 3^4} \times \frac{\log(3)^{7/2}}{\log(2)^2} \times \frac{\log(3)^5}{\log(6)^{7/2}}$$

$$\Rightarrow \frac{7}{2} \times \frac{\log 2}{\log 3} \times \frac{3}{4} \times \frac{\log 6}{\log 3} \times \frac{7}{2} \frac{\log 3}{2 \log 2} \times \frac{5 \log 3}{\frac{7}{2} \log 6}$$

$$= \frac{7}{2} \times \frac{1}{4} \times 7 \times 5$$

$$27^{\log_3 5}$$

$$\Rightarrow (3)^{3 \log_3 5}$$

$$\Rightarrow 3^3 = 125$$

$$2^{\log_3 5} - 5^{\log_3 2}$$

$$\Rightarrow 5^{\log_3 2} - 5^{\log_3 2}$$

$$\Rightarrow 0$$

Prove the following :-

$$\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8} = 0$$

$$\log_2 3 + 2 \times \frac{1}{\log_9 4} - 3 \times \frac{1}{\log_{27} 8} = 0$$

$$\log_2 3 + 2 \log_9 4 + 3 \log_8 27 = 0$$

$$\frac{\log_2 3}{\log_2} + \frac{2 \log_2 4}{2} - \frac{3 \log_2 27}{3} = 0$$

$$\log_2 27 - \log_2 27 = 0$$

$$\frac{\log_3 12}{\log_{36} 3} - \frac{\log_3 4}{\log_{108} 3} = 2$$

$$\Rightarrow \frac{\log_3 12}{1} - \frac{\log_3 4}{\log_3 108}$$

$$\Rightarrow (\log_3 12) (\log_3 36) - (\log_3 4) (\log_3 108)$$

$$\Rightarrow (1 + 2 \log_3 2)(2 + 2 \log_3 2) - (2 \log_3 2)$$

$$\Rightarrow 2 + 6 \log_3 2 + 4 (\log_3 2)^2 - 6 \log_3 2 - 4 (\log_3 2)^2$$

$$\Rightarrow 2$$

If  $a = \log 2$ ,  $b = \log 3$  and  $c = \log 5$   
 then express the following in terms of  $a, b, c$ .

$$(a) \log 12$$

$$\log 48$$

$$\log \left( \frac{25}{25} \right)$$

$$\log 12$$

$$\log (2^2 \times 3)$$

$$\log 2^2 + \log 3$$

$$2 \log 2 + \log 3$$

$$2a + b$$

$$\log 48$$

$$\log (2^2 \times 3 \times 3)$$

$$2 \log 2 + 2 \log 3$$

$$2a + 2b$$

$$4a + b$$